Category of M-algebras and Internal Homomorphisms

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Recall that for (universal) algebras A, B in a category $\mathcal{E}$, $Hom(A, B)$ is usually an external object, in the sense that it is just a set and is neither an algebra nor even an (internal) object of the base category $\mathcal{E}$. Ebrahimi introduced an object $[A, B]$ inside $\mathcal{E}$ to be the best counterpart of $Hom(A, B)$, for universal algebras A, B in a Grothendieck topos $\mathcal{E}$.

In this paper, taking a monoid M and an equational category of algebras $\mathcal{A}$, we introduce the category $Int(\mathbf{M}\mathcal{A})$ of universal algebras in the category $\mathcal{E}$=**MSet**, of sets with an action of a monoid M, together with members of $[A, B]$, called internal homomorphisms, as the set (M-set) of homomorphisms from A to B. We study some algebraic and categoric ingredients of the category $Int(\mathbf{M}\mathcal{A})$, and consider the relation between this category and the category $\mathcal{A}$ as well as the category $M\mathcal{A}$ of algebras in **MSet** with ordinary (external) homomorphisms. Among other things, we show that in this category equalizers do not exist in general while all colimits exist.

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