On closure operators and reflections in Goursat categories

Sandra Mantovani *

By defining a closure operator on effective equivalence relations in a regular category $\mathcal{C}$, it is possible to establish a bijective correspondence between these closure operators and the regular epireflective subcategories $\mathcal{L}$ of $\mathcal{C}$, the same way as Bourn and Gran did for closure operators on kernels in the homological case ([2]). When $\mathcal{C}$ is an exact Goursat category, this correspondence restricts to a bijection between the Birkhoff closure operators on effective equivalence relations and the Birkhoff subcategories of $\mathcal{C}$. In this case it is possible to provide an explicit description of the closure, and we can use this formula to get the closure determined by the reflection of the exact category $\mathcal{T}(\mathbf{HComp})$ of Mal'cev compact Hausdorff algebras into its subcategory $\mathcal{T}(\mathbf{Profin})$ of profinite topological algebras (see [1]). The closure of an equivalence relation S on A is given here by $\overline{S} = S \circ R_A$, where R_A is the congruence on A that identifies two points when they are in the same connected component. Finally, using a recent result of Bourn in [3], it is possible to deduce a characterization of the congruence distributive Goursat categories in terms of a property of the closure operator associated to any Birhkoff subcategory.

References

[1] F. Borceux and M.M. Clementino, *Topological semi-abelian algebras*, Advances Math. 190 (2005) 425-453.

[2] D. Bourn and M. Gran, *Torsion theories in homological categories*, J. Algebra 305 (2006) 18-47.

[3] D. Bourn, *Congruence distributivity in Goursat and Mal'cev categories*, Appl. Categ. Structures, 13 (2005) 101-111.

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