

Extensions in the theory of lax algebras

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Given a unital quantale V and a lax extension \bar{T} to $\text{Mat}(V)$ of a Set-functor T , a natural transformation $\alpha : S \rightarrow T$ induces a lax extension \bar{S} of S as the initial lift of α . When α comes from a monad morphism $\alpha : \mathbb{S} \rightarrow \mathbb{T}$, this process yields a concrete functor $F_\alpha : \text{Alg}(\mathbb{T}, V) \rightarrow \text{Alg}(\mathbb{S}, V)$. In the case where \bar{T} is the Kleisli extension of \mathbb{T} , we put forth sufficient conditions for α to induce an isomorphism. Since $\text{Alg}(\mathbb{T}, V)$ can be obtained as a tower extension of $\text{Alg}(\mathbb{T}, 2)$, this isomorphism is seen to depend essentially on the relation that holds between $\text{Alg}(\mathbb{S}, 2)$ and $\text{Alg}(\mathbb{T}, 2)$.

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