Neighborhoods with respect to a closure operator

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Abstract. Many topological concepts may be defined with the help of the Kuratowski closure operator, without using open sets or neighborhoods. But neighborhoods become indispensable, for instance, if we want to introduce a natural concept of convergence. For this reason, a concept of a neighborhood with respect to a categorical closure operator was defined in [2]. But, in [2], the neighborhoods are not studied in full detail and they are defined under quite a strong assumption that subobject lattices in the category considered are pseudocomplemented.

The aim of the present note is to develop a theory of neighborhoods with respect to a categorical closure operator, without any supplementary assumptions on the category under consideration. Such neighborhoods are obtained by extending the classical neighborhoods in topological spaces in quite a natural way utilizing pseudocomplementation in the subobject lattices. We investigate their basic properties and use them to introduce separation in a natural way. This separation is then studied together with compactness defined in a compatible way. In particular, the separation and compactness are compared with the well-known c-separation and c-compactness [1]. We also introduce and study a natural concept of convergence (of centered classes of subobjects).

REFERENCES

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[2] E. Giuli and J. Šlapal, *Raster convergence with respect to a closure operator*, Cahiers Top. Géom. Différ. Categ. 46 (2006) 275-300.

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