

Profinite relational structures

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Extending Stone duality, Priestley duality is an equivalence between the dual category of distributive lattices (with 0 and 1) and the category of profinite ordered topological spaces, also called Priestley spaces. It is well-known that the Priestley spaces are the compact topological ordered spaces X for which every two distinct points can be separated by a closed-and-open decreasing subset of X . As shown in [2], this can be repeated for preorders, characterizing the profinite preorders.

We show that a preordered topological space is profinite if and only if it is inter-clopen, i.e. it can be presented as an intersection of closed-and-open preorders on the same space. In particular, this gives another characterization of the Priestley spaces, showing that, whenever an ordered topological space can be presented as a limit of finite topologically-discrete preordered spaces it can also be presented as a limit of finite topologically-discrete ordered spaces. But there is no way of considering “order-inter-clopen” simply because there are no such relations.

On the other hand inter-clopen equivalence relations are important: in the case of Stone spaces they are precisely the effective equivalence relations and they were useful in the Galois theory of commutative rings [1], which in fact was our original motivation for considering inter-clopen preorders. Also the effective equivalence relations play an important role in descent theory, namely in the characterization of the effective descent morphisms, as shown in [3].

Furthermore we prove that, under suitable conditions, our main result can be extended from preorders to other relational structures.

REFERENCES

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