Representability Relative to a Doctrine

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By weakening of the representability concept one may obtain, e.g., weaker notions of limits, adjunctions, etc. Various such weak notions are well-known and studied in the literature. We propose the notion of a doctrine that allows to study such weakened representability notions in a uniform way.

By a *doctrine* is understood a pair $(\mathbb{C}, \gamma)$ consisting of a (pseudo)functor $\mathbb{C}$ on CAT and a (pseudo)natural $\gamma : \mathrm{Id} \longrightarrow \mathbb{C}$ such that $\gamma_{\mathscr{X}} : \mathscr{X} \longrightarrow \mathbb{C}(\mathscr{X})$ is fully faithful dense for each category $\mathscr{X}$.

A functor $F : \mathscr{M} \longrightarrow [\mathscr{X}^{op}, \mathsf{Set}]$ is *representable w.r.t.* $(\mathbb{C}, \gamma)$ if it factors to within an isomorphism through the fully faithful $\widetilde{\gamma_{\mathscr{X}}} : \mathbb{C}(\mathscr{X}) \longrightarrow [\mathscr{X}^{op}, \mathsf{Set}]$.

By choosing various doctrines for $(\mathbb{C}, \gamma)$ and various functors for F we obtain various "weak" notions known from the literature: weak limits, plurilimits [2], multi-adjoints [3], and many others.

If the doctrine in question is the doctrine of free cocompletion under a chosen class of colimits, representability can serve as an indication of the existence of certain limits in free cocompletions, [2].

Another advantage of our approach is that one can clearly work with enriched categories instead of "ordinary" ones. In this manner we present, e.g., left coherent rings [1] as a representability notion when the base category is that of Abelian groups.

References

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