Prefactorization systems and well-behaved epireflections

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Let $F \dashv G : \mathbf{X} \to \mathbf{C}$ be an arbitrary adjunction and let $(\mathcal{E}, \mathcal{M})$ be some prefactorization system on $\mathbf{C}$, such that the morphisms in $\mathcal{E}$ are all epimorphisms. If the unit morphisms φ_A of the adjunction $F \dashv G$ do all have an $(\mathcal{E}, \mathcal{M})$-factorization $\varphi_A = \mu_A \circ \eta_A$, for every object A in $\mathbf{C}$, then η is the unit of a full epireflection I from $\mathbf{C}$ into $\{A \in \mathbf{C} \mid \varphi_A \in \mathcal{M}\}$.

The stable units property, for the epireflection so obtained, may be rephrased in this context into easily identifiable necessary and sufficient conditions. Having stable units, there is a simple condition which is enough for obtaining a monotone-light factorization associated with the reflection I. We will derive several examples. In particular, we will give a nice characterization of a class of functors $K : M \to C$ for which the right Kan extensions adjunction $Set^K \dashv Ran_K : Set^M \to Set^C$ always gives rise to an epireflection with stable units and monotone-light factorization.

REFERENCES

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