The Minimal Basis of the Nullspace of Singular Graphs[1]

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Abstract

A minimal basis $\mathcal{B}_{min}$ for the η–dimensional nullspace of the 0–1 adjacency matrix A of a graph G consists of a maximal linearly independent set of vectors with the smallest number of non–zero entries. The support of the vectors in the basis are induced subgraphs called cores. The cores associated with $\mathcal{B}_{min}$ are also induced subgraphs of singular configurations, which are of nullity one and occur as induced subgraphs of G. We show that the nullity controls the maximum core size associated with $\mathcal{B}_{min}$. For a given graph order and maximal nullity, the maximum core size is bounded above. In extremal graphs reaching the upper bound, the maximal core size is shown to be an upper bound of each singular configuration associated with $\mathcal{B}_{min}$. The eigensystem of the 0-1 adjacency matrix of a graph provides sufficient conditions to determine the change in nullity on deleting a vertex v. This can be generalized to the determination of the change in multiplicity of any eigenvalue on deleting a vertex.

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