LIMITS OF INHOMOGENEOUS PROBLEMS RELATED TO THE $p(x)-$LAPLACIAN.

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In this work we study the behaviour of the solutions to inhomogeneous problems related to the $p(x)-$Laplacian operator

$$\Delta_{p(x)}u = -\mathrm{div}(|\nabla u|^{p(x)-2}\nabla u),$$

with Diritchlet and Neumann boundary conditions, as $p(x) \to \infty$. We consider a sequence of functions $p_n(x)$ that goes to infinity uniformly in $\overline{\Omega}$. Under adequate hypotheses on the sequence p_n, basically, that the following two limits exist,

$$\lim_{n\to\infty} \nabla \ln p_n(x) = \xi(x), \quad \text{and} \quad \limsup_{n\to\infty} \frac{\max\limits_{x\in\overline{\Omega}} p_n}{\min\limits_{x\in\overline{\Omega}} p_n} \leq k, \quad \text{for some } \ k>0,$$

we prove that $u_{p_n} \to u_\infty$ uniformly in $\overline{\Omega}$. In addition, we find that u_∞ solves a certain PDE problem (depending on condition) in viscosity sense.

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