Analysis of an economic stochastic differential game with non-market interactions for N players and its finite element approximation

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Abstract

We consider the following systems of N-nonlinear parabolic equations with quadratic growth conditions

$$\partial_t \mathbf{u} - \Delta \mathbf{u} + \mathbf{F}(x, t, \mathbf{u}, \nabla \mathbf{u}) = \mathbf{H}(x, t, \mathbf{u}, \nabla \mathbf{u}) + \mathbf{G}(x, t)$$

which correspond to a stochastic differential games with discount control. It is assumed that $\mathbf{F}$ and $\mathbf{H}$ have quadratic growth in $\nabla \mathbf{u}$. Moreover, $\mathbf{F}$ satisfies some positiveness conditions, that is, $\mathbf{u} \cdot \mathbf{F}(x, t, \mathbf{u}, \nabla \mathbf{u}) \geq 0$ and its Jacobian $\mathbf{F}_{\mathbf{u}}$ is positive definite. In economy and social science the above models strategic situations in which decision makers cannot make binding agreements to co-operate. The solution $\mathbf{u}$ of the system represent a Nash equilibrium point of the corresponding stochastic game. The function $\mathbf{F}$ describes the influence of a player on its own discount factor and the Hamiltonian $\mathbf{H}$ models non-market interaction and the utility function of each player. Recent analytical results are given. A fully discrete Galerkin approximation is proposed using C^0-piecewise linear finite elements in space and a backward Euler time discretization. We present some numerical results. This is a joint work with Carsten Ebmeyer.

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