Application of envelope functions to approximation numbers : two examples

Dorothee D. Haroske (Jena / Germany)

We present some new results on envelope functions in function spaces of type $B^{(s,\Psi)}_{p,q}$ and explain possible applications to approximation numbers. In particular, we obtain for $0 < \sigma < 1$, $0 < q \leq \infty$, $0 < p \leq \infty$, and $s = \frac{n}{p} + \sigma$,

$$\mathcal{E}_{\mathsf{C}}^{B^{(s,\Psi)}_{p,q}}(t) \ \sim \ t^{-(1-\sigma)} \, \Psi(t)^{-1} \ ,$$

where the continuity envelope function of some function space X is defined by

$$\mathcal{E}_{\mathsf{C}}^{X}(t) \ \sim \sup_{\|f|X\| \leq 1} \frac{\omega(f,t)}{t} \ , \quad t > 0,$$

$\omega(f,t)$ being the usual modulus of continuity. This result is used to estimate the approximation numbers of the compact embedding $id : B^{(s,\Psi)}_{p,q}(U) \to C(U)$, $2 \leq p \leq \infty$, $0 < q \leq \infty$, $0 < s - \frac{n}{p} < 1$, Ψ admissible, U the unit ball in $\mathbb{R}^n$, asymptotically by

$$a_k \left(id : B^{(s,\Psi)}_{p,q}(U) \to C(U) \right) \sim k^{-\frac{s}{n} + \frac{1}{p}} \ \Psi\left(k^{-\frac{1}{n}} \right)^{-1} \ , \quad k \in \mathbb{N}.$$

Including an additional lift argument we can similarly determine

$$a_k \left(id : L_p(\log L)_a(U) \to B^{-1}_{\infty,\infty}(U) \right) \sim k^{-\left(\frac{1}{n} - \frac{1}{p}\right)} \, (1 + \log k)^{-a} \ , \quad k \in \mathbb{N},$$

where $n < p < \infty$, $a > 0$.