

The Virtual 10th Combinatorics Day
November 10th and December 7th, 2020

Programme

November 10th, Monday, 2020 – Lisbon/London time (GMT)

17:50–18:00 Get-together

18:00–18:30 **Loïc Foissy (LMPA Joseph Liouville, Université du Littoral Côte d’Opale)**

Algebraic structures on noncrossing partitions

The theory of free probabilities uses the combinatorics of noncrossing partitions in order, for example to establish relations between free cumulants and moments of a random variable. We shall describe the underlying algebraic structures (operads and Hopf algebras).

This a joint work with Kurusch Ebrahimi-Fard, Joachim Kock and Frédéric Patras.

18:40–19:05 **Raul Penaguião (San Francisco State University) :**

Feasible regions meets pattern avoidance - The long awaited part three of feasible regions

Glebbov, Hoppen and others introduced the notion of feasible regions for permutation patterns. Given a fixed integer k , the feasible region is a set in \mathbb{R}^{S_k} defined as follows: for a sequence of permutations σ_k with growing size, compute the limit of the proportion of occurrences of each pattern of size k in σ_n , obtaining a vector indeed by S_n . The feasible region arises as the set of such limits. Many interesting problems were studied in this context, like computing the dimension of the feasible region and its extreme points. Sometimes full descriptions can be given, but an overarching result is missing. If we consider consecutive patterns instead of classical patterns, we get simpler results, and we can characterize the feasible region: it is a polytope, and the vertices are given by cycles of a particular graph called overlap graph. Finally, we will talk about the feasible region for consecutive patterns resulting from considering permutations avoiding certain patterns. These new feasible regions, called the pattern-avoiding feasible regions, are now governed by different versions of the overlap graph, and we propose a unified characterization for all pattern-avoiding feasible regions avoiding one pattern. Along the way, we discuss connections of this work with the problem of packing patterns in pattern-avoiding permutations.

19:15–19:40 **Henrique Cruz (CMA, University of Beira Interior)**

Classes of $(0,1)$ -matrices where the Bruhat order and the Secondary Bruhat order coincide

Given two nonincreasing integral vectors R and S , with the same sum, we denote by $\mathcal{A}(R, S)$ the class of all $(0,1)$ -matrices with row sum vector R , and column sum vector S . The Bruhat order and the Secondary Bruhat order on $\mathcal{A}(R, S)$ are both extensions of the classical Bruhat order on S_n , the symmetric group of degree n . However, these two partial orders on $\mathcal{A}(R, S)$ are, in general, different. In this talk, we describe some classes $\mathcal{A}(R, S)$ where

the Bruhat order and the Secondary Bruhat order coincide. This is joint work with Rosário Fernandes and Domingos Salomão.

19:50–20:15 António Malheiro (CMA, NOVA University of Lisbon)

Crystals & Trees

Kashiwara’s crystal graphs have a natural monoid structure that arises by identifying words labelling vertices that appear in the same position of isomorphic components. The celebrated plactic monoid (the monoid of Young tableaux), arises in this way from the crystal graph for the q -analogue of the general linear Lie algebra gl_n , and the so-called Kashiwara operators interact beautifully with the combinatorics of Young tableaux and with the Robinson–Schensted–Knuth correspondence.

In this talk, we will present an analogous ‘crystal-type’ structure for the Sylvester and Baxter monoids (the monoids of binary search trees and pairs of twin binary search trees, respectively). Both monoids are shown to arise from this structure just as the plactic monoid does from the usual crystal graph. The interaction of the structure with the Sylvester and Baxter versions of the Robinson–Schensted–Knuth correspondence will be also illustrated.

This is joint work with Alan Cain.

December 7th, Monday, 2020 - Lisbon/London time (GMT)

16:50–17:00 Get-together

17:00–17:30 Per Alexandersson (University of Stockholm)

Skew specialized Macdonald polynomials and charge

We study a skew version of a specialization of non-symmetric Macdonald polynomials. These are of similar flavor as the modified Hall-Littlewood polynomials. We describe how to expand these into Schur polynomials using a version of charge. As an application, this gives an explicit Schur-expansion of a class of LLT polynomials, which has previously not been known.

This project is joint work with Joakim Uhlén

17:40–18:05 João Miguel Santos (CMUC, University of Coimbra)

Symplectic right keys - Type C Willis’ direct way

Symplectic keys identify the Demazure crystal atoms which decompose the type C highest weight crystals of Kashiwara-Nakashima tableaux. The right key map on a Kashiwara-Nakashima tableau, a vertex of a such crystal, returns the key tableau that identifies the crystal atom that contains the tableau. The author, following the Lascoux-Schützenberger type A procedure, has shown that the right key of a Kashiwara-Nakashima tableau can be computed

using the Sheats symplectic jeu de taquin. Jacon and Lecouvey reduces the computation of the right key to the fundamental weights and also uses Sheats jeu de taquin. Motivated by Willis' direct way of computing type A right keys, we also give a way of computing symplectic right keys without the use of jeu de taquin. This confirms the Jacon-Lecouvey prediction on a type C direct way.

18:15–18:45 Jake Levinson (Simon Fraser University):

Axioms for shifted tableau crystals

Certain investigations in geometry led Maria Gillespie, Kevin Purbhoo and me to introduce two pairs of raising and lowering operators on shifted semistandard tableaux. These operators give rise to a 'doubled' crystal-like structure on shifted tableaux. This structure recovers the combinatorics of skew Schur Q functions and has applications to the geometry of certain curves in orthogonal Grassmannians.

I will discuss parts of this story, particularly work with Maria Gillespie giving a relatively simple description of these crystals in terms of local graph-theoretic axioms, which closely resemble the Stembridge axioms for crystals in type A.

18:55–19:20 Inês Rodrigues (CEAFEL, University of Lisbon):

Shifted Bender-Knuth moves and a shifted Berenstein-Kirillov group

The Bender-Knuth moves on Young tableaux are well-known involutions, namely, they are used to prove combinatorially the symmetry of Schur functions and are generators for the Berenstein-Kirillov group. They also coincide with the tableau switching on two adjacent letters. Using the shifted tableau switching due to Choi, Nam, and Oh (2019), we introduce a shifted version of the Bender-Knuth operators, which also show the symmetry of P and Q-Schur functions, and define a shifted Berenstein-Kirillov group. This group acts on the straight-shaped shifted tableau crystal introduced by Gillespie, Levinson and Purbhoo (2017), via the partial Schützenberger involutions, thus coinciding with a cactus group action, due to the author. Following the works of Halacheva (2016, 2020), and Chmutov, Glick and Pylyavskyy (2016), we show that the shifted Berenstein-Kirillov group is isomorphic to a quotient of the cactus group, while providing an alternative presentation for the cactus group via shifted Bender-Knuth moves.

19:30–19:55 Maria Manuel Torres (CEAFEL, University of Lisbon):

Eulerian polynomials via the Weyl Algebra action

Through the action of the Weyl algebra on the geometric series, we establish a generalization of the Worpitzky identity and new recursive formulae for a family of polynomials including the classical Eulerian polynomials.

This is a joint work with José Agapito, Pasquale Petrullo and Domenico Senato.

Organizers: Olga Azenhas (CMUC,UC) and Samuel Lopes (CMUP,FC-UP).

URL: <http://www.mat.uc.pt/~combdays/virtual10thcombdays>

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