The Virtual 11th Combinatorics Day-Lisboa
January 21st (afternoon) and January 22nd, 2022

Programme

Day 1: January 21st, Friday, 2022 – Lisbon/London time (GMT)

15:00–15:10 Get-together

15:10–15:50 Iva Halacheva (Department of Mathematics, Northeastern University):

Self-dual puzzles in Schubert calculus and Lagrangian correspondences

In classical Schubert calculus, Knutson and Tao's puzzles give a positive rule for expanding the product of two Schubert classes in equivariant cohomology of the (type A) Grassmannian. I will present a positive rule, using self-dual puzzles, for the pullback of a type A Schubert class to the type C Grassmannian. I will also discuss a generalization in which the Grassmannians are upgraded to their cotangent bundles and Schubert classes to Segre–Schwartz–MacPherson classes. The resulting construction involves Lagrangian correspondences and produces a generalized puzzle rule with a geometric interpretation.

This is joint work with Allen Knutson and Paul Zinn-Justin.

15:55 - 16:20 Maria Dodjig (CEAFEL, University of Lisbon):

Diagrammatics for generalized majorization of integer partitions

The generalized majorization is a generalization of the classical Hardy-Littlewood-Polya majorization for two partitions of integers, to the case when we have three partitions involved. This generalized majorization was motivated by applications in matrix and matrix pencils completion problems, as well as by representation theory of quivers. In this talk we shall describe some of interesting properties of the graphical calculus related to generalized majorization, which are also relevant for applications. In particular, we shall explain how every bottom half of a square diagram can always be completed to a full square, and show some examples of applications of these results.

16:25 - 16:40 Break

16:40 - 17:05 Farrokh Razavinia (CMUP, University of Porto):

A walk through down and up operators in search of deformation

In a 1988 paper, Stanley defined a class of partially ordered sets known as r differential posets. A poset P is called r-differential if every element $x \in P$ is covered by r more elements than x covers. Stanley also defined two operators u (up) and d (down) on elements of P and he showed that these operators satisfy the identity $du - ud = rI$ in any r-differential poset. This concept was expanded upon by Terwilliger who introduced a q-analogue of the differential poset, known as a (q, r)- differential poset. In 1998, in order to extend the ideas of Stanley and Terwilliger, down-up algebras over the complex numbers were introduced by Benkart and Roby and afterwards in 2004, generalized down-up algebras were introduced by Cassidy and Shelton as a generalization

of the down-up algebras. The main part of this talk will be introducing a new class of algebras, which we name quantum generalized Heisenberg algebras (qGHAs, for short), including both the so-called generalized Heisenberg algebras (GHAs, for short) and the generalized down-up algebras and presenting the combinatorial motivation behind our study. We conclude by summarizing our results on the representation theory of qGHAs. We will also present some interesting open directions related to our work.

This is joint work with Professor Samuel Lopes from the University of Porto.

17:10 - 17:50 Thomas Gerber (EPFL, École Polytechnique Fédérale de Lausanne):

Combinatorial Howe duality

The Howe duality is a classical result in Lie theory, which can be expressed as an identity between fundamental tensor multiplicities for $\mathfrak{g}_n$-representations on the one hand and weight multiplicities for $\mathfrak{g}_m$-representations on the other hand. Here $\mathfrak{g}$ is a simple complex Lie algebra of fixed classical type and n, m is the rank of the Lie algebra.

In this talk, I will present a bijective proof of the above identity in the type A and C cases, relying on the combinatorics of tableaux and crystals. More precisely, weight multiplicities are counted by certain tableaux and fundamental tensor multiplicities are counted by sources in certain crystal graphs which we can explicitely put into one-to-one correspondence.

This is joint work with J. Guilhot and C. Lecouvey.

Day2: January 22nd, Saturday, 2022 - Lisbon/London time (GMT)

09:50–10:00 Get-together

10:00–10:40 James Kennedy (GMF, University of Lisbon) :

Spectral graph theory through the eyes of a PDE guy

With any finite graph one can associate, in a natural way, a number of matrices which encode the structure of the graph, such as the Laplacian and the adjacency and incidence matrices. The study of the eigenvalues of such matrices, among other things their relation to geometric properties of the graph, is a well-established topic in graph theory with applications (for example) to the search for "clusters" (highly connected subgraphs) within the given graph.

But one can also study metric graphs, where one identifies each edge with an interval, "glued together" at their endpoints, which then become one-dimensional manifolds with singularities (the vertices). On them one can define, in a natural way, a number of linear differential operators such as the Laplacian, giving rise to so-called "quantum graphs". The study of the eigenvalues of these operators, and in particular their relation to geometric and topological properties of the graph, is a much newer topic, but one which throws up a large number of parallels to its combinatorial counterpart.

We will attempt to give a non-technical introduction to the latter theory, with a particular emphasis on the connections and parallels between difference operators such as the "combinatorial" Laplacian on discrete graphs on the one hand, and differential operators such as the Laplacian on metric graphs on the other. No specialist knowledge of differential equations (or metric graphs) will be assumed.

10:45–11:10 Inês Serôdio Costa (CIDMA, University of Aveiro):

Sharp lower bounds on the least eigenvalue of graphs determined from edge clique partitions

A lower bound on the least eigenvalue of an arbitrary graph is obtained using an edge clique partition and a necessary and sufficient condition for this lower bound be attained is deduced. As an application, we prove that the least eigenvalue of the n-Queens' graph $Q(n)$ is equal to -4 for every $n \geq 4$ and it is also proven that the multiplicity of this eigenvalue is $(n-3)^2$.

Joint work with Domingos M. Cardoso and Rui Duarte.

11:15–11:25 Break

11:25–11:50 Pedro Fernandes (Universität Bonn):

The hidden forest in powers of differential operators

We take a dive into some very particular expressions involving derivatives. However, as we are celebrating Combinatorics, we quickly get out of potentially Analysis filled waters, and find ourselves in the middle of a forest covered with combinatorially flavoured trees.

11:55 - 12:20 Rui Duarte (CIDMA, University of Aveiro):

Pak-Stanley labeling of the Ish arrangement

In the 1990s, Pak and Stanley introduced a construction in which every region of the m-Shi arrangement of hyperplanes is labeled with an m-parking function. In this talk we consider the same construction applied to the regions of the Ish arrangement, a hyperplane arrangement introduced by Armstrong. We characterize the labels of the regions, as well as the labels of the relatively bounded regions. Finally, we present and algorithm for the inverse.

This is joint work with António Guedes de Oliveira.

Organizers: José Agapito (CEAFEL, UL), Olga Azenhas (CMUC, UC), Amélia Fonseca (CEAFEL, UL), Samuel Lopes (CMUP, FCUP), Maria Manuel Torres (CEAFEL, UL).

URL: `http://www.mat.uc.pt/~combdays/virtual11thcombday`

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