The 12th Combinatorics Day-Aveiro
October 21st, 2022

Programme

Room Sala Sousa Pinto
Departamento de Matemática da Universidade de Aveiro

10:00–11:00 Jacinta Torres (Institute of Mathematics, Jagiellonian University in Krakow):

Symplectic cacti, virtualization and colourful reversal of Kashiwara–Nakashima tableaux

I will present an explicit action of the symplectic cactus group, as defined by Halacheva for the Dynkin diagram of a finite root system, on the crystal of Kashiwara–Nakashima tableaux, in terms of the virtualization map due to Baker. Additionally, I will present a "colourful"algorithm which generalizes recent work of Santos' symplectic reversal, in terms of the symplectic jeu de taquin due to Sheats and Lecouvey. We will see many examples along the way. This is joint work with Olga Azenhas and Mojdeh Tarighat Feller.

11:00–11:30 Coffee break

11:30 - 12:00 Ilda P. F. da Silva (University of Lisbon):

Representability of cubes and of qualitative probability orders

Combinatorial Cubes, introduced in [IS08], are matroids $M = M(C^n)$, over the set $C^n := \{0,1\}^n$, that satisfy the following two conditions:

C1) Every set of four 0/1-points that are vertices of a rectangle of $\mathbb{R}^n$ is a circuit of M.

C2) Every set of 0/1 points of $\mathbb{R}^n$ lying on a facet or on a skew facet of the real cube $[0,1]^n$ is a hyperplane and a cocircuit of M.

The class of combinatorial cubes is "huge"but, so far, the real affine cube $Aff(C^n)$ is, up to projective equivalence, the unique known orientable cube. For small dimensions ($n \leq 8$) this is actually true, meaning that linear/affine depencies over $\mathbb{R}$ of 0/1-vectors, with at most 8 entries, can be described in purely combinatorial terms. Recently, together with J. Lawrence [LS21], we proved that the real affine cube is the unique orientable cube that is representable over $\mathbb{R}$.

In this talk I will survey, briefly, results on (orientable) combinatorial cubes and point out some interesting connections with the more "classsic"subject (B. de Finetti 1931) of the representability of qualitative probability orders/boolean term orders.

[IS08] - Ilda P. F. da Silva, "Cubes and Orientability", *Discrete Mathematics*, **308**(2008), 3574-3585.

[L-S21] - J.Lawrence, I.P. Silva, Cubes and adjoints of cross-polytope.

12:00 - 12:30 Jorge Neves (CMUC, University of Coimbra):

Towards the socle of Eulerian ideals

Eulerian ideals were defined in [Neves, Vaz Pinto, Villarreal (2020)]. They are ideals of a polynomial ring with variables indexed by the edges of a simple graph, whose generating set can be described using the Eulerian subgraphs of the graph. By now, several algebraic invariants of Eulerian ideals have been described combinatorially, using the graph. The aim of my talk is to review recent results on the combinatorial characterization of the Hilbert function of Eulerian ideals and also to pose some interesting questions about their socle.

12:30-14:30 Lunch

14:30-15:00 João Queiró (CMUC, University of Coimbra):

Searching for solutions to Horn's problem

Alfred Horn's conjecture on the possible eigenvalues of a sum of Hermitian matrices with given eigenvalues has been proved more than 20 years ago. The question remains of the effective construction of solutions for given spectra. In this talk we present the problem and make some remarks about it.

15:00 - 15:30 Ricardo Guilherme (FCT, Nova University of Lisbon):

Generalizing the hypoplactic monoid through quasi-crystal graphs

A first notion of quasi-crystal graph was proposed by Cain and Malheiro, it was derived from a Kashiwara crystal graph of Cartan type A_n and allowed a construction of the hypoplactic monoid by identifying words in the same position of isomorphic connected components. Cain and Malheiro asked whether this definition could be reformulated to allow a generalization of the hypoplactic monoid as Kashiwara crystals led to a generalization of the plactic monoid, and whether quasi-Kashiwara operators could be defined recursively. In this talk we answer these questions by proposing a notion of quasi-crystals and focus on its combinatorial description that gives rise to a new definition of quasi-crystal graphs. We show that quasi-crystals admit a tensor product that for Cartan type A_n results on a new construction of the hypoplactic monoid. This approach can be applied to other types. Based on this framework, we present some results for the hypoplactic monoid of type C_n.

15:30-16:00 Rute Lemos (CIDMA, University of Aveiro):

On Kubo-Ando and Heinz means of matrices

Spectral inequalities involving Kubo-Ando operator connections and means of positive semidefinite matrices are considered. Some of these inequalities are of log-majorization type. Singular value inequalities for the Heinz mean of matrices, which are not of Kubo-Ando type, and its harmonic variant are also presented. This talk is based on joint work with Graça Soares (CMat-UTAD).

16:00 - 16:30 Coffee

16:30 - 17:30 Thomas Gerber (EPFL, École Polytechnique Fédérale de Lausanne):

Looking for cores

It is well-known that the combinatorics of core partitions plays an essential role in block theory for symmetric groups and related algebraic structures. Motivated by these representation-theoretic interpretations, Granville and Ono

proved in 1996 the following celebrated result: there exists an e-core of every size as soon as $e \geq 4$. In this talk, we are interested in analogous results for different generalisations of cores. We will report on work in progress with Emily Norton and with Nathan Chapelier.

17:30-18:00 Henrique Cruz (CMA, University of Beira Interior):

Efficient Vectors for Simple Perturbed Consistent Matrices

An $n \times n$ matrix $A = [a_{ij}]$ is said to be a pairwise comparison matrix (brieffy a PC-matrix) if it is a positive matrix and

$$a_{ij} = \frac{1}{a_{ji}},$$

for all $i, j = 1, \ldots, n$. If, in addition, $a_{ij}a_{jk} = a_{ik}$, $i, j, k = 1, \ldots, n$, then A is said to be consistent or transitive. PC matrices, particularly consistent ones, play an essential role in the Analytic Hierarchy Process. This mathematical tool helps decision makers to choose the best option in a problem that evolves a finite number of alternatives. In this method, it may be essential to approximate a PC matrix by a consistent one. In this context, the notion of an efficient vector for a PC matrix arises. A weight vector is efficient for a given pairwise comparison matrix if no other weight vector is at least as good in approximating the matrix elements and strictly better in at least one position. Describing the efficient vectors for an arbitrary pairwise comparison matrix is a problem that is still open. This talk describes all efficient vectors for a simple perturbed consistent matrix. These matrices differ from consistent ones in just one entry above the main diagonal and its reciprocal.

This is a joint work with Rosário Fernandes and Susana Furtado.

Organizers: Olga Azenhas (CMUC, UC), Rui Duarte (CIDMA, UA), Samuel Lopes (CMUP, FCUP).

URL: `http://www.mat.uc.pt/~combdays/12thcombday`

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