The 13th Combinatorics Day-Covilhã
July 13-14, 2023

Programme

Departamento de Matemática da Universidade da Beira Interior

Day 1: July 13th, Thursday, 2023

9:30–11:00 Sándor Bozóki (Institute for Computer Science and Control, SZTAKI, Budapest) :

Spanning trees in the graph of pairwise comparisons I

Pairwise comparisons are frequently applied in multi-criteria decision making and ranking. The set of known comparisons can be complete or incomplete. Vitaliy Tsyganok's works (since 2000) drew attention to the significance of the edge-minimal, connected sets, i.e., spanning trees in the graph of comparisons. Earlier works focused on particular spanning trees, such as the star graph. The mini-course presents some, mostly recent results on the weight vectors calculated from sets of comparisons, corresponding to spanning trees. The weight vector calculated from a complete or incomplete pairwise comparison matrix by the logarithmic least squares method, can also be written as the geometric mean of weight vectors, calculated from the comparisons associated to the spanning trees of the comparisons' graph. The Laplacian matrix plays the main role in the proof.

Furthermore, for any number N of edges, larger than the number of items minus two, the weight vector calculated from a complete pairwise comparison matrix by the logarithmic least squares method, can also be written as the geometric mean of weight vectors, calculated from the comparisons associated to the connected subgraphs with E edges. The proof is illustrated by a special symmetry in the GRAPH of connected labelled graphs (there is an EDGE between two graphs if one of them can be given by the addition of one edge to the other graph). Spanning trees are at the top, the complete graph is at the bottom of this multi-partite GRAPH. However, the proposition does not extend to the incomplete case due to a broken symmetry.

Simulations show that a star graph is optimal among (now unlabelled) spanning trees if we compare the corresponding weight vectors to the weight vector calculated from the complete pairwise comparison matrix. However, as we increase the number of edges, more and more regular graphs appear among the optimal graphs.

Pareto optimality (efficiency) of a weight vector is a natural requirement. Once we can find a better (dominating) weight vector, which estimates all the matrix elements at least as good, and strictly better in at least one position, it is hard to argue with the use of the dominated one. A weight vector calculated from a spanning tree's comparisons is always Pareto optimal. Most results are presented within the framework on the multiplicative/additive/reciprocal pairwise comparison matrix format, used in the Analytic Hierarchy Process related literature. However the direct relation to physical realizations such as electric circuits is also highlighted.

11:00–11:30 Coffee break

11:30 - 13:00 Jake Levinson (Simon Fraser University):

Flag Varieties and Schubert Calculus I

An introduction to flag varieties and the combinatorics of Schubert polynomials, Schur polynomials and Young tableaux.

I will plan to discuss a combination of the following:

- the Schubert stratification of the complete flag variety (and perhaps a shorter discussion of the Grassmannian).
- the cohomology ring of the flag variety, in terms of the Schubert polynomials. Monk's rule.
- the Schur polynomials – Pieri rule, Littlewood-Richardson rule.
- some open problems.

13:00-15:00 Lunch

15:00 - 15:30 Claudio Piedade (CMUP, University of Porto):

Core-free Degrees of Toroidal Maps

Every group G can be represented as a faithful transitive permutation representation of degree n. Moreover, the stabilizer of a point in this permutation representation is always a core-free subgroup of G. Conversely, the action of a group G on a core-free subgroup $H \leq G$ is always transitive and faithful, giving a faithful transitive permutation representation on the set of cosets G/H, with degree $|G : H|$. These permutation representations are powerful tools in the classification of abstract regular/chiral polytopes and hypertopes. In this talk we list all possible degrees of faithful transitive permutation representations of the toroidal regular/chiral maps $\{4, 4\}$, $\{3, 6\}$ and hypermaps $(3, 3, 3)$. This is a joint work with M. Elisa Fernandes.

15:30-16:00 Diogo Soares (CMUC, University of Coimbra) :

The commutation graph for the longest signed permutation

Using the standard Coxeter presentation for the signed symmetric group on n+1 letters, two reduced expressions for a given signed permutation are in the same commutation class if one expression can be obtained from the other one by applying a finite sequence of commutations. The commutation classes of a given signed permutation can be seen as the vertices of a graph, called the commutation graph, where two classes are connected by an edge if there are elements in those classes that differ by a long braid relation. We study some properties of this graph for the longest signed permutation. In particular, we compute its radius and diameter, study its plannarity and compute the number of commutation classes that have only one element. An application to visualize and explore properties of these graphs for certain values of n will also be presented.

16:00 - 16:30 Rúben Palma (Nova University of Lisbon):

How we can make use of directed Hamiltonian graphs to spot a efficient vector for a pairwise comparison matrix

Abstract: In this presentation, I will be talking about the technique I devolped together with Rosário Fernandes on how to approach the problem of characterizing efficient vectors for pairwise comparison matrices with a certain type of perturbation, and how this method could be useful for future problems related with the Analytic Hierarchy Process.

16:30–17:00 Coffee break

17:00-17:30 Inês Rodrigues (FCT, Nova University of Lisbon):

A local characterization of quasi-crystal graphs

The hypoplactic monoid (the monoid of quasi-ribbon tableaux), first introduced by Krob and Thibon (1997) and studied in depth by Novelli (2000), is an analogue of the classical plactic monoid (the monoid of Young tableaux), appearing in the context of quasi-symmetric functions and non-commutative symmetric functions. Cain and Malheiro (2017) introduced a purely combinatorial quasi-crystal structure for the hypoplactic monoid, where words are equal in the monoid precisely when they are in the same position in isomorphic connected components of a quasi-crystal graph.

We provide a local characterization of quasi-crystal graphs, by presenting a set of local axioms, similar to the ones introduced by Stembridge (2003) for crystal graphs of type A (or, more generally, for simply-laced root systems). We then prove that quasi-crystal graphs satisfying these axioms are closed under the tensor product recently introduced by Cain, Guilherme and Malheiro (2023) and conclude that each connected component of such a graph has a unique highest weight element, whose weight is a composition, and it is isomorphic to a quasi-crystal graph of semistandard quasi-ribbon tableaux.

This is joint work with Alan Cain, António Malheiro and Fátima Rodrigues.

17:30-18:00 Neeraj Kumar (CMUP, University of Porto):

On counting numerical semigroups

Numerical semigroups are the cofinite subsemigroups of the set of natural numbers together with the identity element. We discuss the notion of counting numerical semigroups through different invariants (such as the genus and the Frobenius number). We discuss their properties, and the key problems associated with them. Then we introduce a new way of counting, namely by fixing the maximum generator, and discuss its properties. We show that this new way of counting is, in some sense, equivalent to the one associated with the Frobenius number.

18:00-18:30 Inês Legatheaux Martins (FCUL):

The rank partition and partial symmetries on tensors

In 1990, J. A. Dias da Silva introduced a matroid invariant known as the rank partition. For matroids realizable over the complex numbers, this combinatorial concept relates matroid theory to the representation theory of the symmetric group S_n. In particular, it plays a crucial rôle in the context of the annulment problem of a symmetrized decomposable tensor.

In this talk, we turn our attention to the rook monoid R_n. Also known as the symmetric inverse monoid, this algebraic structure captures partial symmetry much in the same way as the symmetric group captures symmetry. Since its representation theory is built upon that of symmetric groups, it provides a natural context to generalize concepts such as symmetry classes of tensors and symmetrized tensors.

The aim of this talk is to define partial symmetry classes of tensors, state some vanishing problems for partially symmetrized decomposable tensors and prove that the rank partition also plays a crucial part in this context. Our results

suggest some interesting questions and applications of the rank partition that have a strong combinatorial flavour.

Day2: July 14th, Friday, 2023

09:30–11:00 Jake Levinson (Simon Fraser University) :

Flag Varieties and Schubert Calculus II

11:00–11:30 Coffee break

11:30–13:00 Sándor Bozóki (Institute for Computer Science and Control, SZTAKI, Budapest):

Spanning trees in the graph of pairwise comparisons II

13:00–14:30 Lunch:

14:30–15:00 David Correia (ETH Zürich):

Hamilton cycles in pseudorandom graphs

Finding general conditions which ensure that a graph is Hamiltonian is a central topic in graph theory. An old and well known conjecture in the area states that any d-regular n-vertex graph G whose second largest eigenvalue in absolute value $\lambda(G)$ is at most d/C, for some universal constant $C > 0$, has a Hamilton cycle. We obtain two main results which make substantial progress towards this problem. Firstly, we settle this conjecture in full when the degree d is at least a small power of n. Secondly, in the general case we show that $\lambda(G) \leq d/C(\log n)^{1/3}$ implies the existence of a Hamilton cycle, improving the 20-year old bound of $d/(\log n)^{1-o(1)}$ of Krivelevich and Sudakov. We use in a novel way a variety of methods, such as a robust Pósa rotation-extension technique, the Friedman-Pippenger tree embedding with rollbacks and the absorbing method, combined with additional tools and ideas. Our results have several interesting applications, giving best bounds on the number of generators which guarantee the Hamiltonicity of random Cayley graphs, which is an important partial case of the well known Hamiltonicity conjecture of Lovász. They can also be used to improve a result of Alon and Bourgain on additive patterns in multiplicative subgroups. This is joint work with Stefan Glock and Benny Sudakov.

15:00 - 15:30 Gonçalo Varejão (CMUC, University of Coimbra):

On the generators of the Eulerian ideal of a graph

In the past decades, a popular trend in commutative algebra has been the investigation of homogeneous ideals, in polynomial rings over a field, that are defined using simple graphs, and the relations between their properties and the properties of the graphs. For the first part of this talk, we review some invariants studied in this context, namely, the Castelnuovo-Mumford regularity of powers of ideals.

Then, we focus on one particular ideal, associated with a simple graph, the Eulerian ideal. This is a homogeneous binomial ideal, and its generators

depend on the Eulerian subgraphs, of the graph, with even number of edges. Motivated by the first part of the talk, we present some properties on the generators of this ideal.

15:30 - 16:00 Manuel Silva (FCT, Nova University of Lisbon):

Distant words

It is known that any infinite aperiodic word in a finite alphabet has at least n+1 distinct factors of length n (factor complexity theorem, Thue 1906). Our aim is to prove that, for essentially all infinite binary words, it is possible to find two consecutive factors whose distance is large. We will consider two different notions of distance, the most appropriate for our problem will be invariant under cyclic translation. We will discuss results of a similar nature, when we have more than two consecutive factors.

Organizers: Olga Azenhas (CMUC, UC), Henrique Cruz (CMA, UBI), Ilda Inácio (CMA, UBI), Samuel Lopes (CMUP, FCUP).

URL: `http://www.mat.uc.pt/~combdays/13thcombday.html`

Sponsors: Departamento de Matemática da Universidade da Beira Interior, CMA, CMUC, CMUP and FCT.