The 14th Combinatorics Days-Almada
June 27-29, 2024

Programme

Day 1: June 27th, Thursday, 2024

9:00–9:15 Reception

9:15–9:30 Opening session

9:30–11:00 Mercedes Rosas (University of Sevilla):

Vector partition function in representation theory I

Focusing on the chamber complex of $SU(3)$ and the study of the linear symmetries of its tensor multiplicities, we illustrate how vector partition functions appear in representation theory. We assume no prior knowledge of representation theory or vector partition functions, and dedicate the first two lectures to these classic topics.

11:00–11:30 Coffee break

11:30 - 12:00 Carlos André (CMUC, CMAFcIO, University of Lisbon):

Extreme supercharacters of the infinite unitriangular group

We define extreme supercharacters of the infinite unitriangular group, introduce the ramification scheme associated with the classical supercharacter theories of the finite unitriangular groups and describe how extreme supercharacters of the infinite unitriangular group appear as weak limits of supercharacters of those. In order to prove that the set of extreme supercharacters is closed (with respect to the topology of weak convergence), we will deform the ramification scheme using appropriate operations of induction and restriction, and show that the resulting scheme is multiplicative and determines a convenient Riesz ring.

12:00-12:30 Tânia Silva (NOVA SBE):

TBA

12:30-13:00 Rui Duarte (CIDMA, University of Aveiro):

Decomposition of Rectangles in Dominos

In 1961, Temperley & Fisher and Kasteleyn independently determined that the number of domino tilings of an $k \times \ell$ rectangle is given by

$$T(\ell,k) = \prod_{p=1}^{\ell}\prod_{q=1}^{k} \sqrt[4]{4\cos^2\left(\frac{p\,\pi}{\ell+1}\right) + 4\cos^2\left(\frac{q\,\pi}{k+1}\right)}.$$

Using determinants and a natural decomposition we prove this formula in a simple way.

Joint work with António Guedes de Oliveira.

13:00-15:00 Lunch

15:00 - 16:30 **Travis Scrimshaw (Hokkaido University):**

Mini-course: (K-theoretic) Schubert calculus and stochastic processes I

In this minicourse, we will discuss the interrelations between three classical subjects: Schubert calculus, the boson-fermion correspondence with solvable lattice models, and the totally asymmetric simple exclusion process (TASEP).

We will begin with Schubert calculus; although our treatment will be mostly combinatorial, we will briefly discuss some of the algebraic geometry underlying as motivation. We will start with cohomology and move to K-theory. Next, we will translate our symmetric functions into more algebraic information by using the famed boson-fermion correspondence between representations originating in mathematical physics, allowing us to derive some identities.

We will also discuss how this is related with another concept from physics that is inherently combinatorial, solvable lattice models. A slight digression will be made to give an application to alternating sign matrices. Lastly, we will introduce the TASEP in a few different variations, survey a few results, and show that the K-theoretic symmetric functions and the boson-fermion correspondence encodes the information of these particle processes. Throughout this minicourse, we will mention a number of open problems.

16:30-17:00 **Coffee break**

17:00 - 17:30 **Oscar Morales (CMUP):**

Twisted Gelfand-Tsetlin modules

In this presentation, we will discuss combinatorial methods of constructing strongly tame Gelfand-Tsetlin modules for $\mathfrak{sl}_n$. We obtain large family of simple modules that have a basis consisting enumerable bases of Gelfand − Tsetlin tableaux where the action of the Lie algebra is given by the twisted Gelfand-Tsetlin formulas. As an application of these construction which show that in the principal and minimal nilpotent orbits of $\mathfrak{sl}_n$, all simple induced weight modules of some $\mathfrak{sl}_2$-cuspidal are admissible as representation in the vertex algebra.

References

1) J. C. Arias, O. Morales, and L. E. Ramirez, Combinatorial Proprieties of Relation Gelfand-Tsetlin modules, in progress.

2) V. Futorny, O. A. Hernández Morales and L. Křižka, Admissible representations of simple affine vertex algebras, J. Algebra, (2023).

3) V. Futorny, O. A. Hernández Morales and L. E. Ramirez. Simple modules for affine vertex algebras in the minimal nilpotent orbit. IRMN, (2021).

4) K. Kawasetsu, Relaxed highest-weight modules III: Character formulae, Adv. Math., 393 (2021), Paper No. 108079.

5) K. Kawasetsu and D. Ridout, Relaxed highest weight modules II: Classification for affine vertex algebras, Commun. Contemp. Math., (2021).

17:30 - 18:00 **Carlos Florentino (CMAFcIO, FCUL):**

Constructing polygon spaces with prescribed topology

A polygon space is a space that parametrizes all spatial polygons (in $\mathbb{R}^3$) with sides of given fixed lengths (up to rigid motions). For a generic polygon with $n > 3$ sides, this space is a smooth symplectic (in fact, complex projective) manifold of real dimension $2n - 6$. Its topology has been computed by A.

Klyachko in a famous paper, where he gives a formula for its Betti numbers. In this talk, I will focus on a construction question: given a sequence of Betti numbers, can we find a polygon space with that topology? We settle it for n up to 6 (hexagons), but the general question remains open. We show that this problem is related to the famous subset-sum problem, and to McMullen's "upper bound theorem"for polytopes.

18:00-18:30 Ricardo Mamede (CMUC, University of Coimbra):

The commutation classes of a permutation

Any permutation w of the symmetric group can be generated by a product of adjacent transpositions, and a reduced word for w is a sequence of generators of minimal length whose product is w. We consider the graph $G(w)$ whose vertices are reduced words for w and whose edges are braid relations. We establish a statistic on the set of reduced words for w, inducing a rank poset structure on the graph $G(w)$. This statistic allows to prove a conjecture made by Reiner and Roichman on bounds for the diameter of $G(w)$, and to compute the diameter of $G(w)$ for certain permutations w. To do so, we define a metric on the set of all reduced words of a given permutation which turn out to be equal to the usual distance in any commutation class. If a permutation is fully commutative, i.e. if it has only one commutation class, then the formula gives the diameter of $G(w)$. The diameter for a Grassmanian permutation is also given in terms of its Lehman code.

Day 2: June 28th, Friday, 2024

09:30–11:00 Travis Scrimshaw (Hokkaido University):

Mini-course: (K-theoretic) Schubert calculus and stochastic processes II

11:00–11:30 Coffee break

11:30–13:00 Mercedes Rosas (University of Sevilla):

Mini-course: Vector partition function in representation theory II

13:00–15:00 Lunch:

15:00–16:00 Travis Scrimshaw (Hokkaido University):

Mini-course: (K-theoretic) Schubert calculus and stochastic processes III

16:00 - 16:30 Rosário Fernandes (NOVA Math, NOVA FCT):

Some partial orders on the class of $(0,1)$*-matrices and related conjectures*

Let R and S be two sequences of positive integers in nonincreasing order having the same sum. Let $A(R,S)$ be the class of $(0,1)$-matrices with row sum R and column sum S. The Bruhat order and the secondary Bruhat order are two partial orders defined on $A(R,S)$. The minimal matrices for the Bruhat order on $A(R,S)$ are minimal matrices for the secondary Bruhat order. In 2004, it was conjecture that the converse is true. In this talk, we will see results and conjectures related with this.

References

[1] R. Fernandes, H. F. da Cruz, D. Salomão. *On a conjecture concerning the Bruhat order.* Linear Algebra and its Applications 600 (2020) 82-95.

[2] R. A. Brualdi, S. G. Hwang. *A Bruhat order for the class of* $(0,1)$*-matrices with row sum vector* R *and column sum vector* S*.* Electronic journal of Linear Algebra 12 (2004) 6-16.

16:30-17:00 Coffee break

17:00 - 17:30 Leonardo Saud Maia Leite (KTH Royal Institute of Technology):

On chain polynomials of posets

The chain polynomial of a finite lattice $\mathcal{L}$ is given by $p_{\mathcal{L}}(x) = \sum_{k\geq 0} c_k(\mathcal{L})x^k$, where $c_k(\mathcal{L})$ is the number of chains of length k in $\mathcal{L}$. The chain polynomials of posets in several important classes have been proven to be real-rooted, for example face lattices of simplicial and cubical polytopes, and $(3+1)$-free posets, proved by Brenti and Welker, Athanasiadis, and Stanley, respectively. However, Stembridge presented some distributive lattices that do not have real-rooted polynomials. Recently, Athanasiadis and Kalampogia-Evangelinou conjectured that the chain polynomials of geometric lattices are always real-rooted. We present a sufficient theorem that proves this conjecture for the lattice of flats of paving matroids and generalized paving matroids. We also apply this theorem to other posets to verify that their chain polynomials are also real-rooted. This is a joint work with Petter Brändén.

17:30 - 18:00 Tânia Paulista (NOVA Math, NOVA FCT):

Commuting graph of a 0*-Rees matrix semigroup over a group*

The commuting graph of a finite non-commutative semigroup S is a simple graph whose vertices are the non-central elements of S, and where two distinct vertices $x, y \in S$ are adjacent if and only if $xy = yx$.

We study the commuting graph of a 0-Rees matrix semigroup over a group, and determine some of the properties of this graph – connectedness, diameter, clique number, chromatic number, girth and knit degree.

The interest of studying 0-Rees matrix semigroups over groups is justified by the Rees–Suschkewitsch theorem, which provides a characterization of completely 0-simple semigroups: a semigroup is completely 0-simple if and only if it is isomorphic to a 0-Rees matrix semigroup over a group. This way, studying the commuting graph of a 0-Rees matrix semigroup over a group will provide us with information regarding the properties of commuting graphs of completely 0-simple semigroups.

Day 3: June 29th, Saturday, 2024

09:30–10:30 Mercedes Rosas (University of Sevilla):

Mini-course: Vector partition function in representation theory III

10:30 - 11:00 Inês Costa (CIDMA, University of Aveiro):

A new family of regular integral graphs

An integral graph is a graph whose spectrum is entirely integer. The study of integral graphs was introduced by F. Harary and A. J. Schwenk in [3], where they question "Which graphs have integral spectra?", with the remark that the problem for general graphs appears intractable. As it is referred in [1], the number of these graphs is infinite, and they appear in all classes of graphs and among graphs of all orders. Despite the existence of trivial examples of integral graphs, as it is the case of complete graphs, they are very rare and difficult to be found [1]. In [1] and [4], a survey of results on integral graphs is presented. In this talk, a new family of integral (and regular) graphs will be introduced. The graphs of this family have a spectrum with a consistent structure and eigenvectors with very fascinating patterns. These and other properties of these graphs will be presented. Joint work with Domingos M. Cardoso1 and Rui Duarte.

[1] K. Balinska, D. Cvetkovi, Z. Radosavljevi, S. Simi, and D. Stevanovi. A survey on integral graphs. Publ. Elektrotehn. Fak. Ser. Mat, 13: 42-65, 2002. URL: https://api.semanticscholar.org/CorpusID:11047869

[2] D. M. Cardoso, I. S. Costa, R. Duarte. Sharp bounds on the least eigenvalue of a graph determined from edge clique partitions, J Algebr Comb, 58: 263- 277, 2023. DOI:10.1007/s10801-023-01247-1

[3] F. Harary and A. J. Schwenk. Which graphs have integral spectra? Graphs and Combinatorics, 45-51. Springer Berlin Heidelberg, 1974.

DOI: 10.1007/BFb0066434

[4] L. Wang. A survey of results on integral trees and integral graphs. Number 1763 in Memorandum Afdeling TW. University of Twente, 2005.

11:00–11:30 Coffee break

11:30 - 12:30 Persi Diaconis (Stanford University):

An Introduction to Computational Polya Theory

We all know there are $n^{(n-2)}$ rooted labeled trees on n points. But, there is no simple formula for the number of unlabeled trees. More generally, If X is a finite set and G is a group acting on X, describing the orbits, counting them and asking about their sizes can be challenging problems. Classical Polya theory contributes to this via cycle generating functions. BUT computer scientists Leslie Goldberg and Mark Jerum have started to harness these tools to help with enumeration. I will introduce the Burnside process and show that it leads to nice algorithms and math problems. As a special case, I'll study *how can we generate a random integer partition of* n, when say n is a million and we want a lot of them (special case of the symmetric group acting on itself by conjugation) . Of course, sometimes these problems are provably intractable and this raises the problems *how do you show a problem is difficult*?

17:30 - 18:00 Manuel Branco (University of Évora):

Arithmetic varieties of numerical semigroups

In this talk, we present the notion of arithmetic variety for numerical semigroups. We study various aspects related to these varieties such as the smallest arithmetic that contains a set of numerical semigroups and we exhibit the root three associated with an arithmetic variety. This tree is not locally finite. However, if the Frobenius number is fixed, the tree has many nodes and algorithms can be developed. Joint work with Ignacio Ojeda (Universidad de Extremadura) and J.C.Rosales (Universidad de Granada).

Organizers: Olga Azenhas (CMUC, UC), Samuel Lopes (CMUP, FCUP), Claudio Piedade (CMUP, FCUP), Fátima Rodrigues (NOVA Math, NOVA FCT), Inês Rodrigues (NOVA Math, NOVA FCT).

URL: http://www.mat.uc.pt/ combdays/14thcombday.html

Sponsors: NOVA FCT, NOVA Math, CMUC, CMUP, CIM and FCT.