Locating domination in graphs

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All the graphs considered are finite, undirected, simple, and connected. The vertex set and edge set of a graph G are denoted by $V(G)$ and $E(G)$. The distance between vertices $v, w \in V(G)$ is denoted by $d(v, w)$.

Definition 1. *A set D of vertices in a graph G is a* dominating set *if every vertex of $V(G) \setminus D$ has a neighbour in D. The* domination number $\gamma(G)$ *is the minimum cardinality of a dominating set of G.*

Definition 2. *A set $D = \{x_1, \ldots, x_k\}$ of vertices in a graph G is a* locating set *if for every pair of distinct vertices $u, v \in V(G)$, $(d(u, x_1), \ldots, d(u, x_k)) \neq (d(v, x_1), \ldots, d(v, x_k))$. The* location number $\beta(G)$ *is the minimum cardinality of a locating set of G.*

Definition 3. *An η-*code *is a locating dominating set of minimum cardinality. The* metric-location-domination number $\eta(G)$ *is the cardinality of an η-code of G.*

Definition 4. *A set D of vertices in a graph G is a* locating-dominating set *if for every two vertices $u, v \in V(G) \setminus D$, $\emptyset \neq N(u) \cap D \neq N(v) \cap D \neq \emptyset$. A λ-*code *is a locating-dominating set of minimum cardinality. The* location-domination number $\lambda(G)$ *is the cardinality of a λ-code of G.*

As a straightforward consequence of these definitions, the following inequalities hold:

Proposition 1. *For every graph G,* $\max\{\gamma(G), \beta(G)\} \leq \eta(G) \leq \min\{\gamma(G) + \beta(G), \lambda(G)\}$

In this talk, we will present the state of the art, including some of our contributions, concerning η-codes and λ-codes. To be more precise, we will discuss and provide details mainly, but not only, regarding the results appearing in Table 1 and Theorem 1.

G	γ	β	η	λ
P_2	1	1	1	1
P_3	1	1	2	2
P_n, $n > 3$	$\lceil \frac{n}{3} \rceil$	1	$\lceil \frac{n}{3} \rceil$	$\lceil \frac{2n}{5} \rceil$
C_4, C_5	2	2	2	2
C_6	2	2	3	3
C_n, $n > 6$	$\lceil \frac{n}{3} \rceil$	2	$\lceil \frac{n}{3} \rceil$	$\lceil \frac{2n}{5} \rceil$
K_n, $n > 1$	1	$n-1$	$n-1$	$n-1$
$K_{1,n-1}$, $n > 2$	1	$n-2$	$n-1$	$n-1$
$K_{r,n-r}$, $r > 1$, $n > 4$	2	$n-2$	$n-2$	$n-2$
$W_{1,4}$	1	2	2	2
$W_{1,5}$	1	2	3	3
$W_{1,6}$	1	3	3	3
$W_{1,n-1}$, $n > 7$	1	$\lfloor \frac{2n}{5} \rfloor$	$\lceil \frac{2n-2}{5} \rceil$	$\lceil \frac{2n-2}{5} \rceil$

Table 1: Domination parameters of some basic families

Theorem 1. *Let G be a graph such that $|V(G)| = n$, $diam(G) = D \geq 3$, $\eta(G) = \eta$ and $\lambda(G) = \lambda$. Then*

- $\eta(G) + \lceil \frac{2D}{3} \rceil \leq n \leq \eta + \eta \cdot 3^{\eta-1}$*, and both bounds are tight.*
- $\lambda + \lfloor \frac{3D+1}{5} \rfloor \leq n \leq \lambda + 2^{\lambda} - 1$*, and both bounds are tight.*
- *Let $h \in \{1, 2, n-2, n-1\}$. The set of all graphs of order $n \geq 2$ satisfying $\eta(G) = h$ (resp. $\lambda(G) = h$) has been completely characterized.*
- *Given three positive integers a, b, c verifying that $\max\{a, b\} \leq c \leq a + b$, there always exists a graph G such that $\gamma(G) = a$, $\beta(G) = b$ and $\eta(G) = c$, except for the case $1 = b < a < c = a + 1$.*

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