On direct enumeration of permutominoes

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A polyomino is an edge-connected set of unit squares on a regular square lattice. It is said to be row-convex (column-convex) if all its rows (columns) are connected sets, and *convex* if it is both row-convex and column-convex. Although the exact counting and enumeration of polyominoes remains an open problem, several positive results were achieved for special classes of polyominoes, namely for the class of convex polyominoes and some of its subfamilies (e.g., directed-convex polyominoes, parallelogram polyominoes, stack polyominoes, and Ferrers diagrams) and also the larger class of row-convex (or column convex) polyominoes. These classes, which satisfy convexity and/or directness conditions, have been studied using different approaches and are fairly well characterized, for some parameters, e.g., area and perimeter [1].

Permutominoes were introduced in [4, 3]. A permutomino of size $r+1$ is a simply-connected polyomino determined by a pair (π_1, π_2) of permutations of size $r+2$, such that $\pi_1(i) \neq \pi_2(i)$, for all $1 \leq i \leq r+2$. Its boundary is an orthogonal polygon with r reflex vertices (and, therefore, $2r+4$ vertices). This polygon has exactly one edge in each line of the $(r+1) \times (r+1)$ minimal bounding box it. As for generic polyominoes, also some subclasses of permutominoes satisfying convexity and/or directness conditions have been studied. In [2] an algorithm is given for the exhaustive generation of the convex ones.

In an independent work, Tomás and Bajuelos introduced the class of permutominoes as the relevant class for the generation of generic orthogonal polygons without holes [5], calling them *grid orthogonal polygons*, and developed two construction techniques – INFLATE-CUT and INFLATE-PASTE – for their generation. In this talk, I will explain these techniques and describe a direct enumeration algorithm for generic permutominoes based on this approach. This characterization allows us to understand the structure of some subclasses of permutominoes, count some of them, and solve art gallery problems in some of the restricted subclasses efficiently.

References

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