A tour through spherical f-tilings

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We are interested in the study of a class of tilings of the sphere, spherical f-tilings.

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Summary

We are interested in the study of a class of tilings of the sphere, spherical f-tilings.

Spherical f-tilings are strongly related to the theory of isometric foldings:

S. A. Robertson, Isometric foldings of Riemannian manifolds, *Proceedings of the Royal Society of Edinburgh* **79** (1977) 275–284.

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Isometric folding

Let M and N be smooth Riemannian manifolds. A map

$$f : M \longrightarrow N$$

is an isometric folding if f sends finite piecewise geodesic segments to finite piecewise geodesic segments of the same length.

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Let M and N be smooth Riemannian manifolds. A map

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Let $\mathcal{F}(M, N)$ be set of all isometrics foldings from M into N. Then,

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Let $\mathcal{F}(M, N)$ be set of all isometrics foldings from M into N. Then,

i) $\mathcal{F}(M) = \mathcal{F}(M, M)$ is a semigroup with identity element id_M and contains the isometry group $\mathcal{I}(M)$ as a sub-semigroup;

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Let $\mathcal{F}(M, N)$ be set of all isometrics foldings from M into N. Then,

i) $\mathcal{F}(M) = \mathcal{F}(M, M)$ is a semigroup with identity element id_M and contains the isometry group $\mathcal{I}(M)$ as a sub-semigroup;

ii) for all x, $y \in M$, $d_N(f(x), f(y)) \leq d_M(x, y)$, where d_M and d_N are, respectively, the induced metrics on M and N by their Riemannian structure. And so any isometric folding is a continuous map;

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i) $\mathcal{F}(M) = \mathcal{F}(M, M)$ is a semigroup with identity element id_M and contains the isometry group $\mathcal{I}(M)$ as a sub-semigroup;

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iii) any differentiable isometric folding is an isometry.

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Σf - The set of all singularities of f (points where f fails to be differentiable).

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Σf - The set of all singularities of f (points where f fails to be differentiable).

An isometric folding f is said non trivial if $\Sigma f \neq \varnothing$.

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Σf - The set of all singularities of f (points where f fails to be differentiable).

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A general description of Σf, for any $f \in \mathcal{F}(M, N)$, was given by S. A. Robertson in 77.

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Let M and N be complete Riemannian 2-manifolds and let $f \in \mathcal{F}(M, N)$.

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Description of Σf for surfaces

Let M and N be complete Riemannian 2-manifolds and let $f \in \mathcal{F}(M, N)$.

The singularities of f near x ($x \in \Sigma f$) form the image of an even number of geodesic rays emanating from x and making alternated angles $\alpha_1, \beta_1, \alpha_2, \beta_2, \ldots, \alpha_n, \beta_n$, where

$$\sum_{j=1}^{n} \alpha_j = \sum_{j=1}^{n} \beta_j = \pi. \tag{1}$$

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$$\sum_{j=1}^{n} \alpha_j = \sum_{j=1}^{n} \beta_j = \pi. \qquad (1)$$

The singularity set of an isometric folding on surfaces can be seen as an embedded graph of even valency at any vertex and satisfying the angle folding relation (1).

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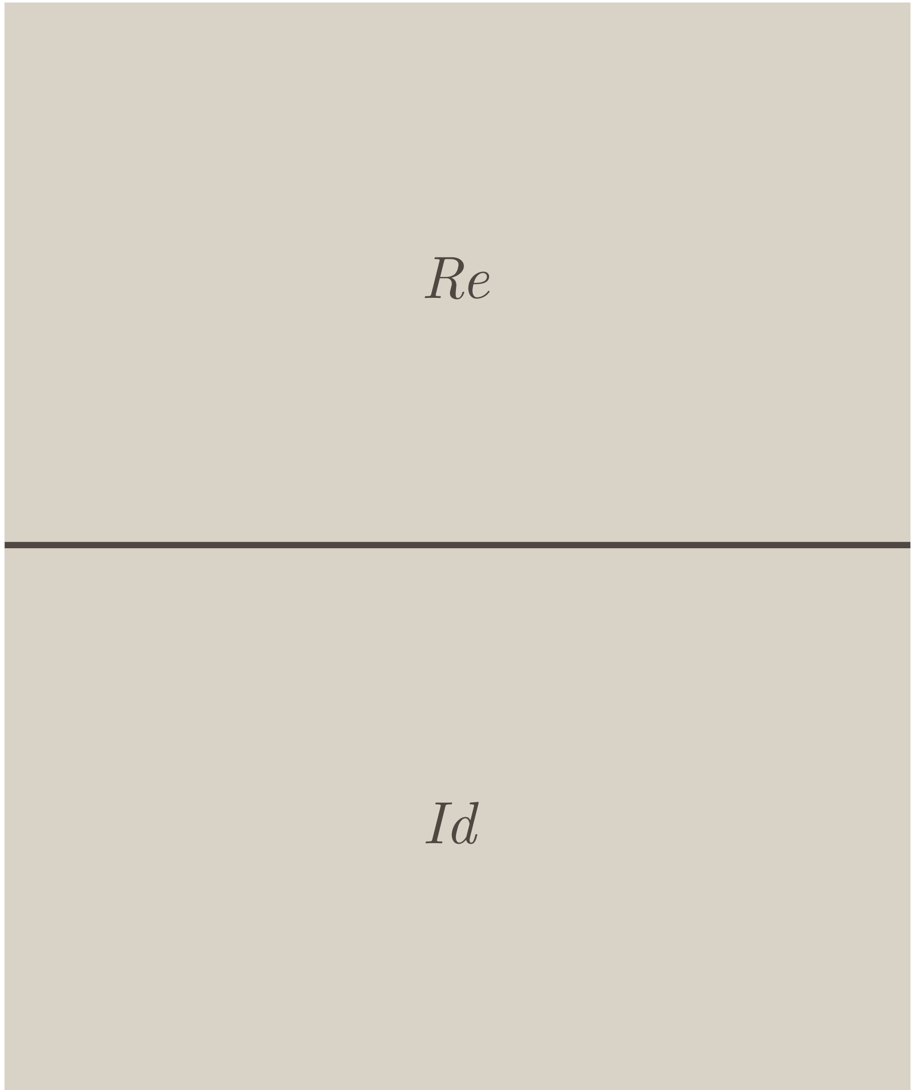

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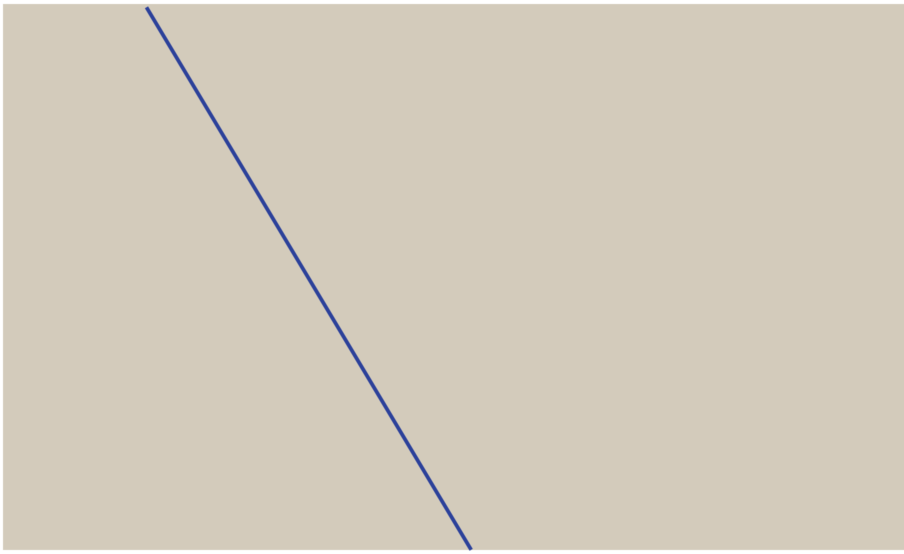

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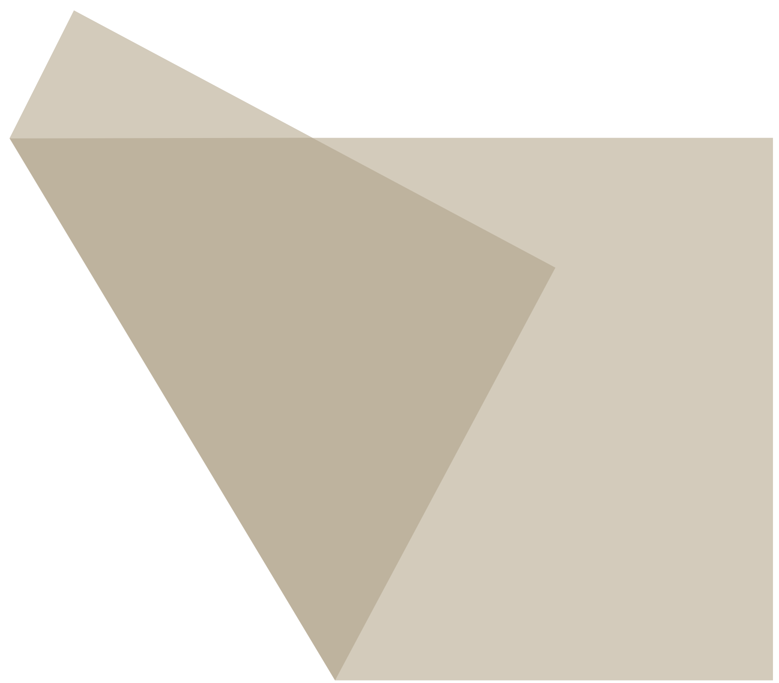

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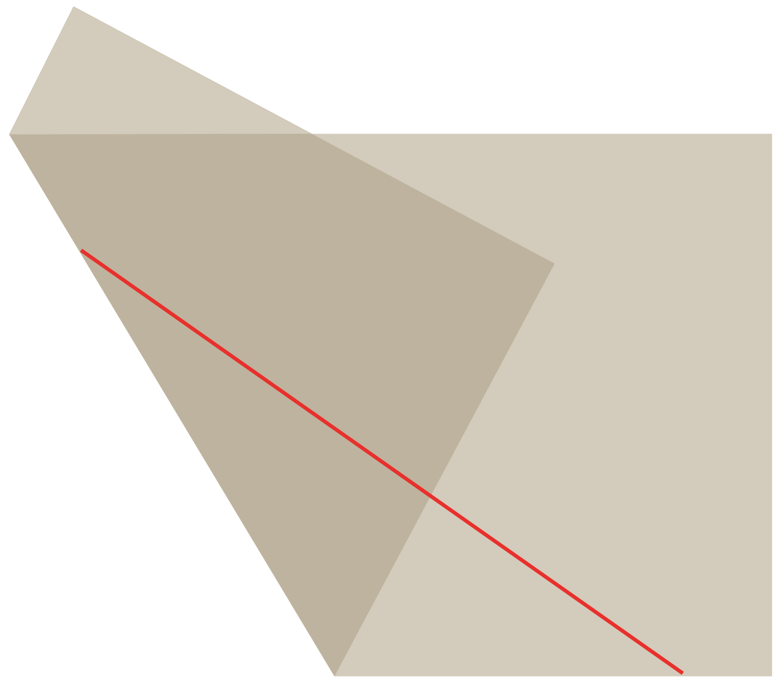

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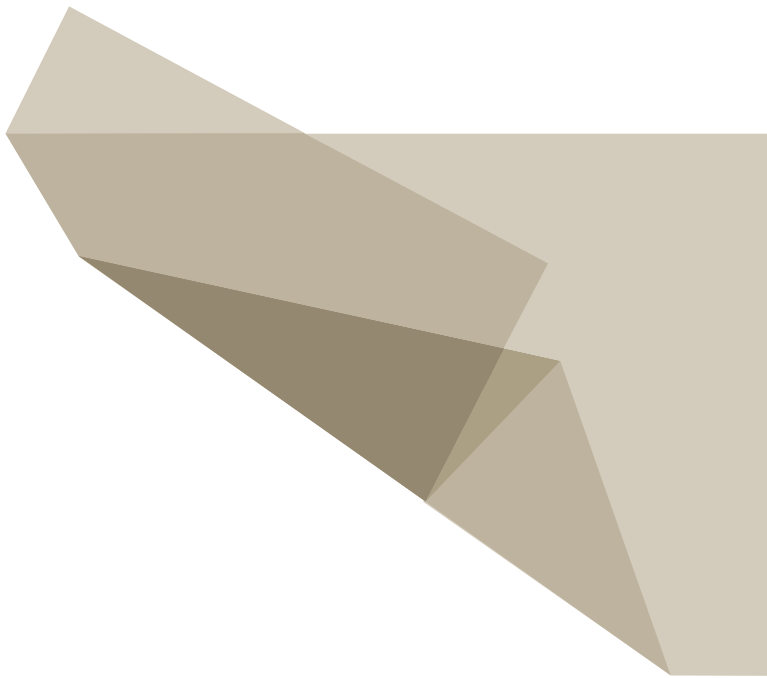

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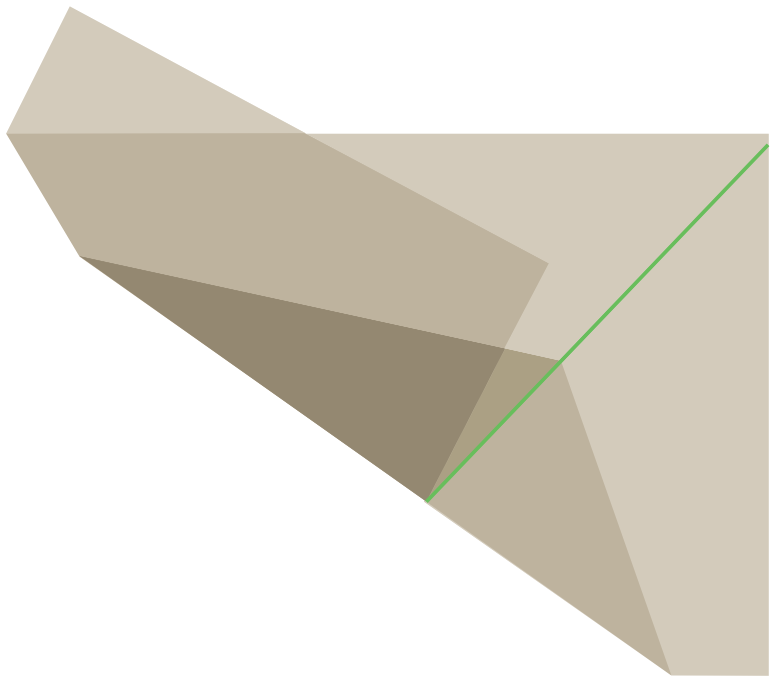

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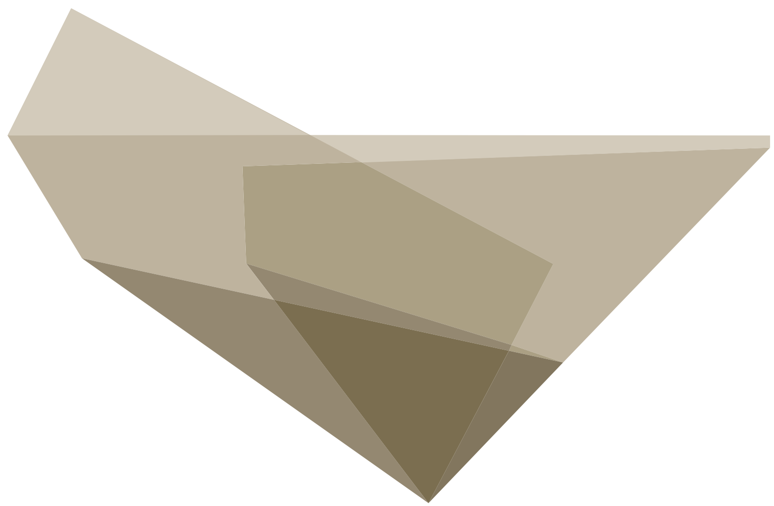

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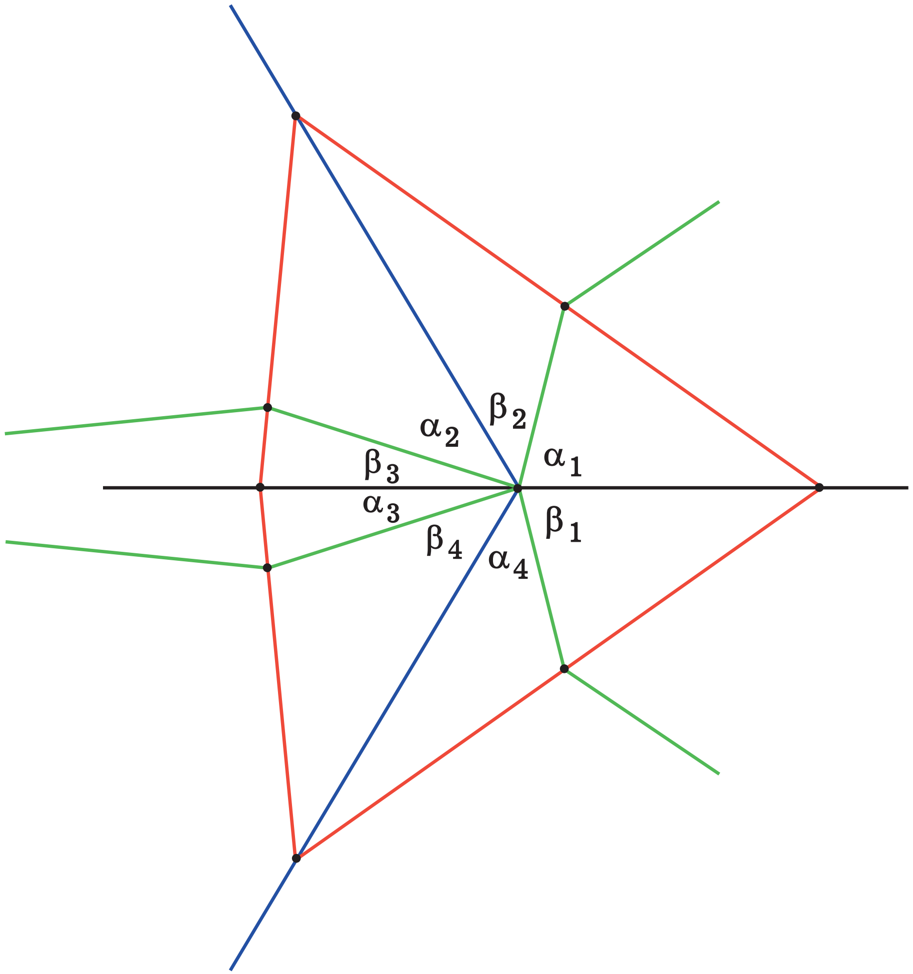

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The compactness of the sphere assures that the singularity set of any spherical isometric folding is connected with finitely many regions.

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The compactness of the sphere assures that the singularity set of any spherical isometric folding is connected with finitely many regions.

A spherical folding tiling is an edge-to-edge finite polygonal-tiling τ of S^2 whose underlying graph is of the type described in 1.

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The compactness of the sphere assures that the singularity set of any spherical isometric folding is connected with finitely many regions.

A spherical folding tiling is an edge-to-edge finite polygonal-tiling τ of S^2 whose underlying graph is of the type described in 1.

We shall denote by $\mathcal{T}(S^2)$ the set of all folding tilings of S^2 identifying the singularity sets of non-trivial foldings with spherical folding tilings.

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Classification of spherical folding tilings with a specified fixed type of prototiles:

1. Classification of all spherical monohedral f-tilings, A. M. Breda, 1992.

   obs: The prototile must be a spherical triangle since any (convex) polyhedron in $\mathbb{R}^3$ must have, at least, a triangular face or a vertex of valency 3.

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   Ten years later was established the complete classification of all triangular spherical monohedral tilings. (which obviously includes the monohedral f -tilings). Y. Ueno, Y. Agaoka - 2002.

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- Dihedral spherical f-tilings:

  Triangle $+$ **Triangle**

  Triangle $+$ **Quadrangle**

  Triangle $+$ **Convex Polygon**

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- Dihedral spherical f-tilings:

  Triangle + Triangle

  Triangle + Quadrangle

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2. Classification of all dihedral spherical f-tilings by triangles and parallelograms, A. M. Breda, A. F. Santos - from 2004 to 2006.

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  2.2 Dihedral f-tilings by spherical triangles and spherical rhombi.

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   2.2 Dihedral f-tilings by spherical triangles and spherical rhombi.

   2.3 Dihedral f-tilings by spherical triangles and spherical parallelograms with distinct pairs of congruent opposite angles and with distinct pairs of congruent opposite sides.

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3. Classification of all dihedral f-tilings of the sphere by triangles and r-sided regular polygons ($r \geq 5$), C. P. Avelino, A. F. Santos - 2008.

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4. Classification of all dihedral triangular f-tilings of the sphere whose prototiles are an equilateral triangle and any other triangle,

   A. M. Breda, P. S. Ribeiro and A. F. Santos - from 2008 to 2009.

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4. Classification of all dihedral triangular f-tilings of the sphere whose prototiles are an equilateral triangle and any other triangle, A. M. Breda, P. S. Ribeiro and A. F. Santos - from 2008 to 2009.

5. Classification of all dihedral f-tilings of the sphere by isosceles trapezoids and (equilateral and isosceles) triangles, C. P. Avelino, A. F. Santos - 2011.

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6. Classification of all dihedral f-tilings of the sphere by isosceles trapezoids and scalene triangles, C. P. Avelino, A. F. Santos - from 2009 to 2012.

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7. Classification of the dihedral f-tilings of the sphere by two right triangles being one of each isosceles, C. P. Avelino, A. F. Santos - 2012.

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7. Classification of the dihedral f-tilings of the sphere by two right triangles being one of each isosceles, C. P. Avelino, A. F. Santos - 2012.

8. Classification of the dihedral f-tilings of the sphere by any two isosceles triangles, A. M. Breda, P. S. Ribeiro - in work.

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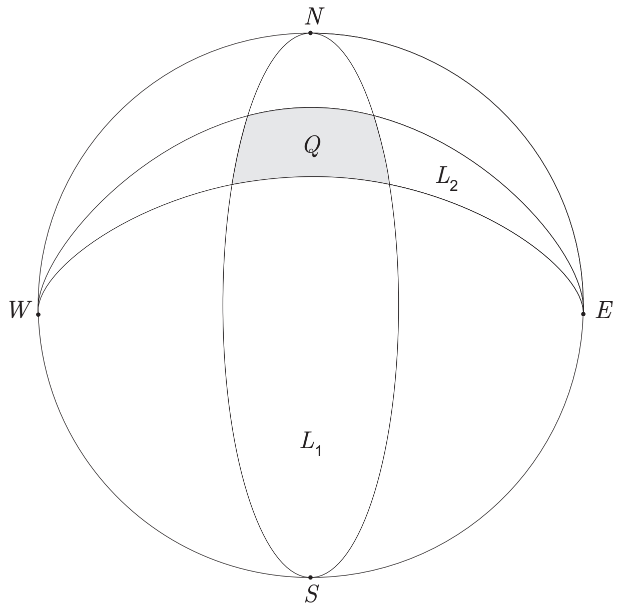
Let S^2 be the Euclidean sphere of radius 1. A spherical isosceles trapezoid is a spherical quadrangle congruent to the intersection of two spherical lunes, $Q = L_1 \cap L_2$, where L_1 and L_2 have vertices in the plane $x = 0$, in orthogonal positions, and L_1 has the point $(1, 0, 0)$ at its center.



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Let $\Omega(Q,T)$ be the set, up to an isomorphism, of all dihedral f-tilings of S^2 whose prototiles are an isosceles trapezoid Q and an isosceles triangle T.

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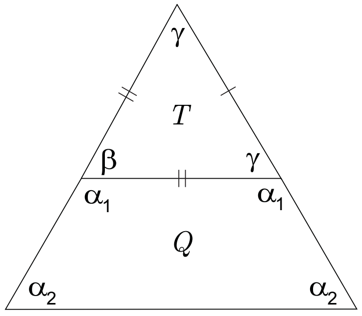

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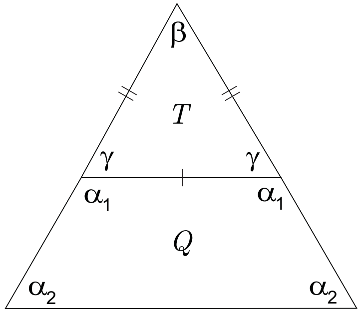

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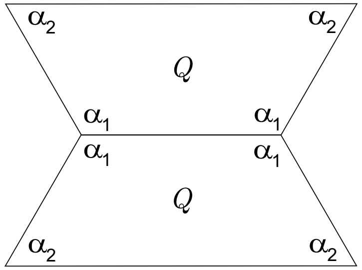

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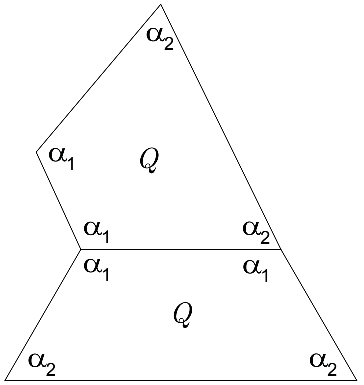

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Proposition 1. *If* $\Omega(Q,T) \neq \emptyset$ *then there cannot be a pair of tiles as in cases I or IV.*

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Proposition 2. *Let Q and T be a spherical isosceles trapezoid and a spherical isosceles triangle, respectively, such that they are in adjacent positions as in case II. Then, $\Omega(Q,T) \neq \emptyset$ iff*

(i) $\alpha_1 + \beta = \pi$ *and* $\alpha_2 = \gamma = \frac{\pi}{2}$ *or*

(ii) $\alpha_1 + \gamma = \pi$, $2\alpha_2 + \gamma = \pi$ *and* $\beta = \frac{\pi}{2}$ *or*

(iii) $\alpha_1 + \gamma = \pi$, $\alpha_2 = \frac{\pi}{3}$ *and* $\beta = \frac{\pi}{2}$ *or*

(iv) $\alpha_1 + \gamma = \pi$, $\alpha_2 = \frac{\pi}{2}$ *and* $\beta = \frac{\pi}{k}$, *for some* $k \geq 2$.

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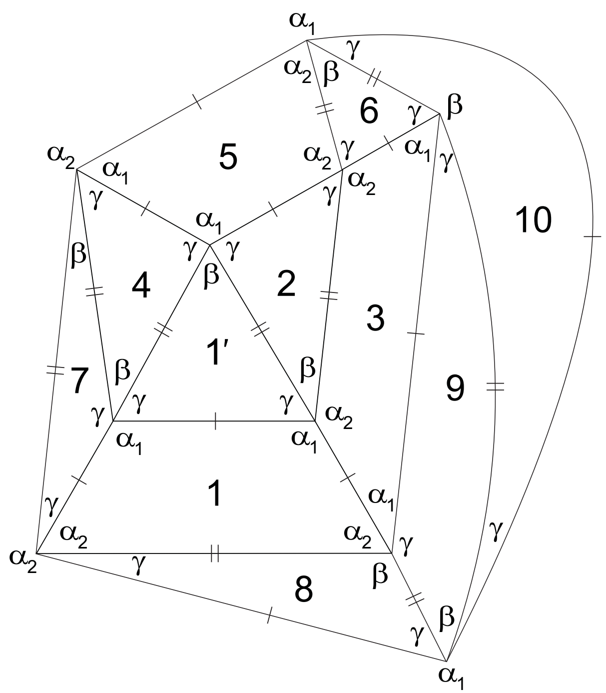

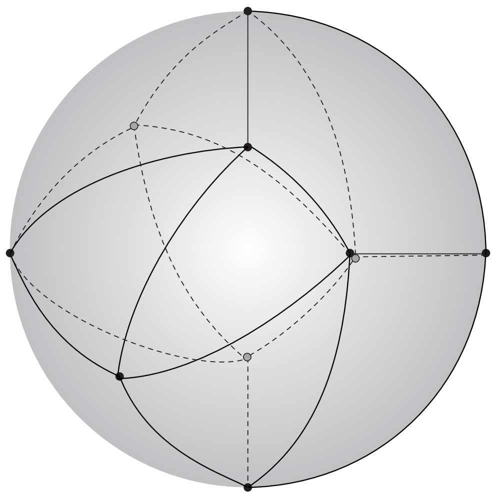

Figure 1: f-tiling $\mathcal{T}$

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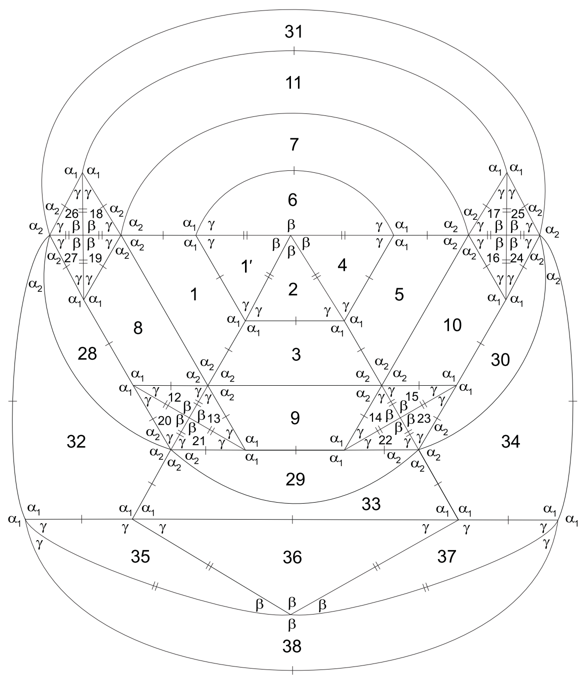

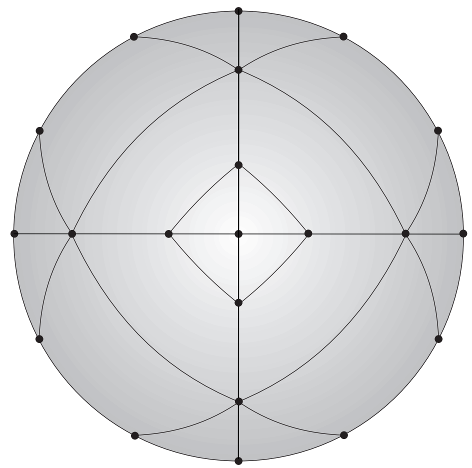

Figure 2: f-tiling $\mathcal{C}$

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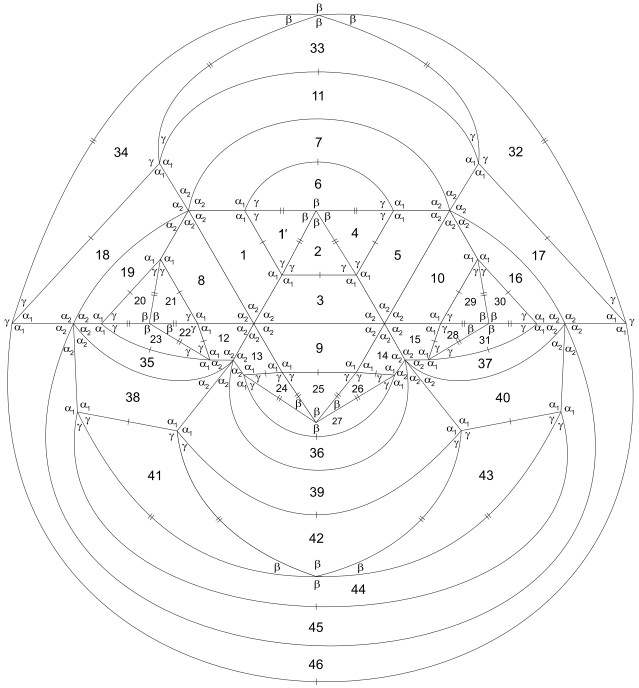
(iii) $\alpha_1 + \gamma = \pi$, $\alpha_2 = \frac{\pi}{3}$ *and* $\beta = \frac{\pi}{2}$:



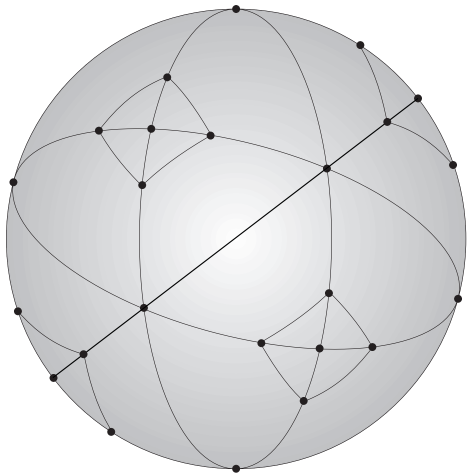


Figure 3: *f-tiling* $\bar{\mathcal{C}}_\gamma$, $\gamma \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

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(iv) $\alpha_1 + \gamma = \pi$, $\alpha_2 = \frac{\pi}{2}$ *and* $\beta = \frac{\pi}{k}$, *for some* $k \geq 2$*:*

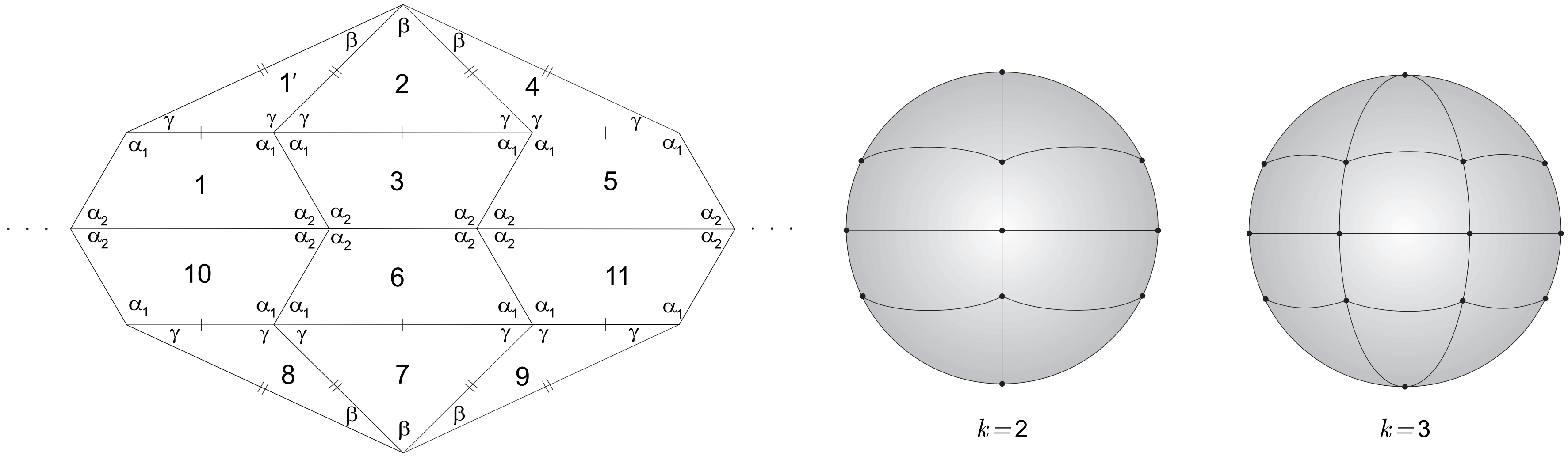

Figure 4: f-tiling $\mathcal{R}^k_{\alpha_1}$, $k \geq 2$ and $\alpha_1 \in \left(\frac{\pi}{2}, \frac{(k+1)\pi}{2k}\right)$

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Proposition 3. *If two spherical isosceles trapezoids are in adjacent positions as in case III, then* $\Omega(Q,T) = \left\{\bar{\mathcal{R}}_{\alpha_2}^k \mid k \geq 3\right\} \cup \{\mathcal{C}\}$*, where* $\bar{\mathcal{R}}_{\alpha_2}^k$ *is a dihedral f-tiling satisfying* $\alpha_1 + \gamma = \pi$*,* $\alpha_2 + \beta = \pi$ *and* $\gamma = \frac{\pi}{k}$*, with* $\alpha_2 \in \left(\pi - \arccos\left(-\cos^2\frac{\pi}{k}\right), \frac{2\pi}{k}\right)$ *and* $k \geq 3$*.*

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Proposition 3. *If two spherical isosceles trapezoids are in adjacent positions as in case III, then* $\Omega(Q,T) = \left\{\bar{\mathcal{R}}^k_{\alpha_2} \mid k \geq 3\right\} \cup \{\mathcal{C}\}$, *where* $\bar{\mathcal{R}}^k_{\alpha_2}$ *is a dihedral f-tiling satisfying* $\alpha_1 + \gamma = \pi$, $\alpha_2 + \beta = \pi$ *and* $\gamma = \frac{\pi}{k}$, *with* $\alpha_2 \in \left(\pi - \arccos\left(-\cos^2 \frac{\pi}{k}\right), \frac{2\pi}{k}\right)$ *and* $k \geq 3$.

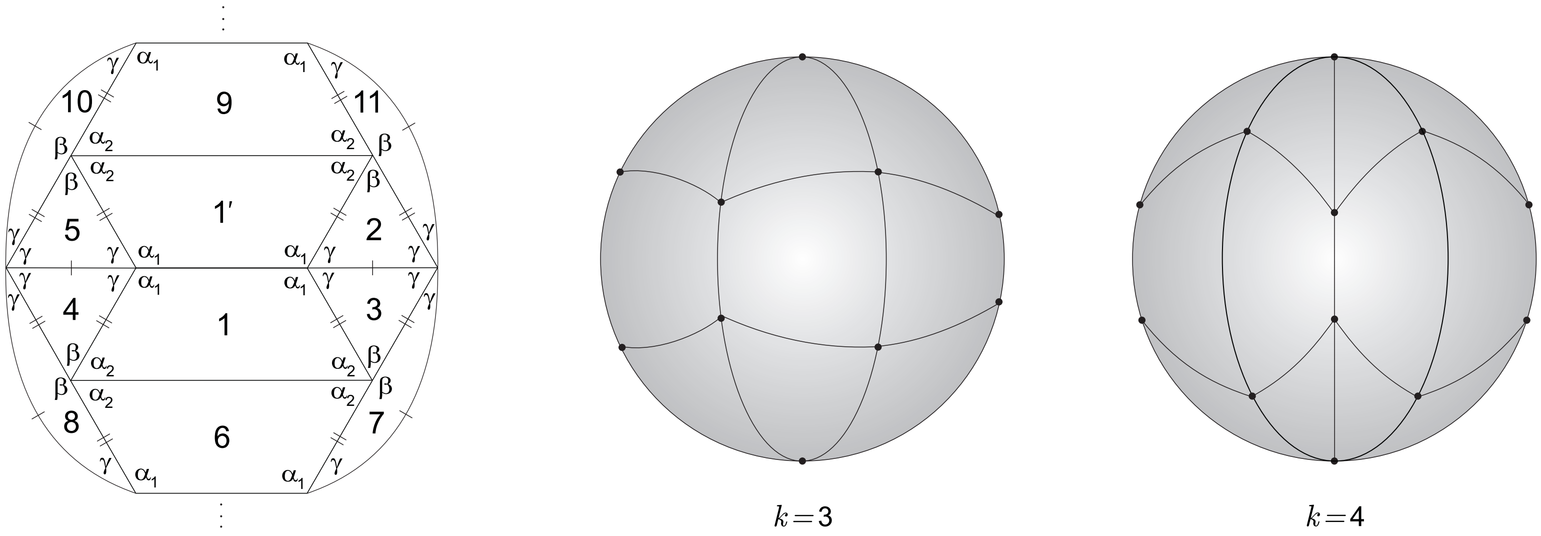

Figure 5: *f-tiling* $\bar{\mathcal{R}}^k_{\alpha_2}$, $k \geq 3$ *and* $\alpha_2 \in \left(\pi - \arccos\left(-\cos^2 \frac{\pi}{k}\right), \frac{2\pi}{k}\right)$

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How to obtain the f-tilings?

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How to obtain the f-tilings?

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(i) Consider restrictions over Q and T;

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How to obtain the f-tilings?

An “algorithm”...

(i) Consider restrictions over Q and T;

(ii) common vertex of Q and T (adjacents);

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How to obtain the f-tilings?

An “algorithm”...

(i) Consider restrictions over Q and T;

(ii) common vertex of Q and T (adjacents);

(iii) list of possible vertices; remove those that do not allow the local configuration give rise to a global configuration;

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How to obtain the f-tilings?

An “algorithm”...

(i) Consider restrictions over Q and T;

(ii) common vertex of Q and T (adjacents);

(iii) list of possible vertices; remove those that do not allow the local configuration give rise to a global configuration;

(iv) build a planar representation;

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An “algorithm”...

(i) Consider restrictions over Q and T;

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(iii) list of possible vertices; remove those that do not allow the local configuration give rise to a global configuration;

(iv) build a planar representation;

(v) complete analysis of all the angles and edges; study the congruence of them;

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An “algorithm”...

(i) Consider restrictions over Q and T;

(ii) common vertex of Q and T (adjacents);

(iii) list of possible vertices; remove those that do not allow the local configuration give rise to a global configuration;

(iv) build a planar representation;

(v) complete analysis of all the angles and edges; study the congruence of them;

(vi) build a 3D representation.

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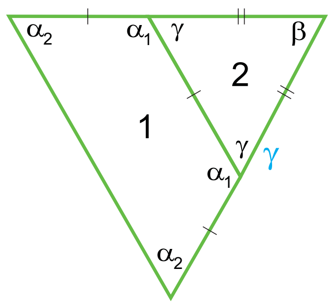

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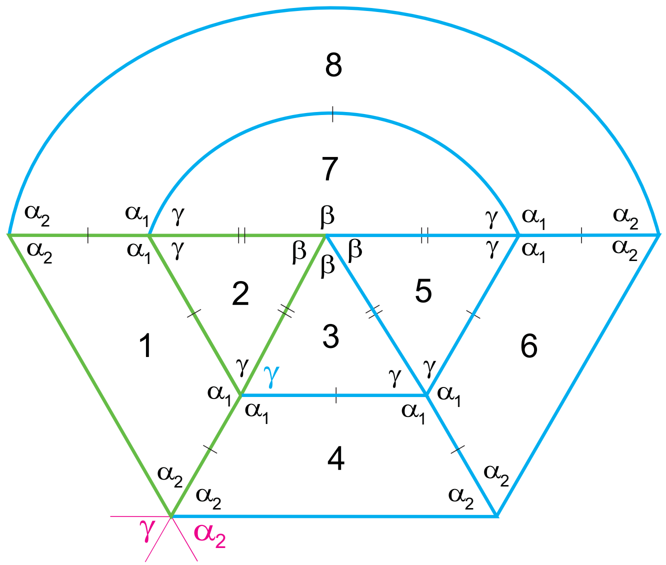

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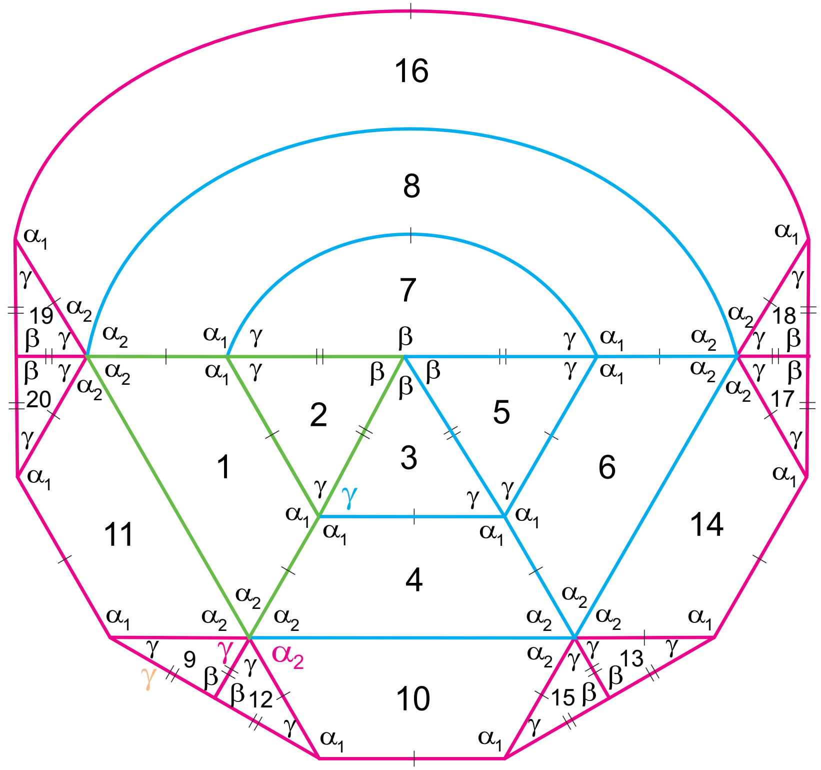

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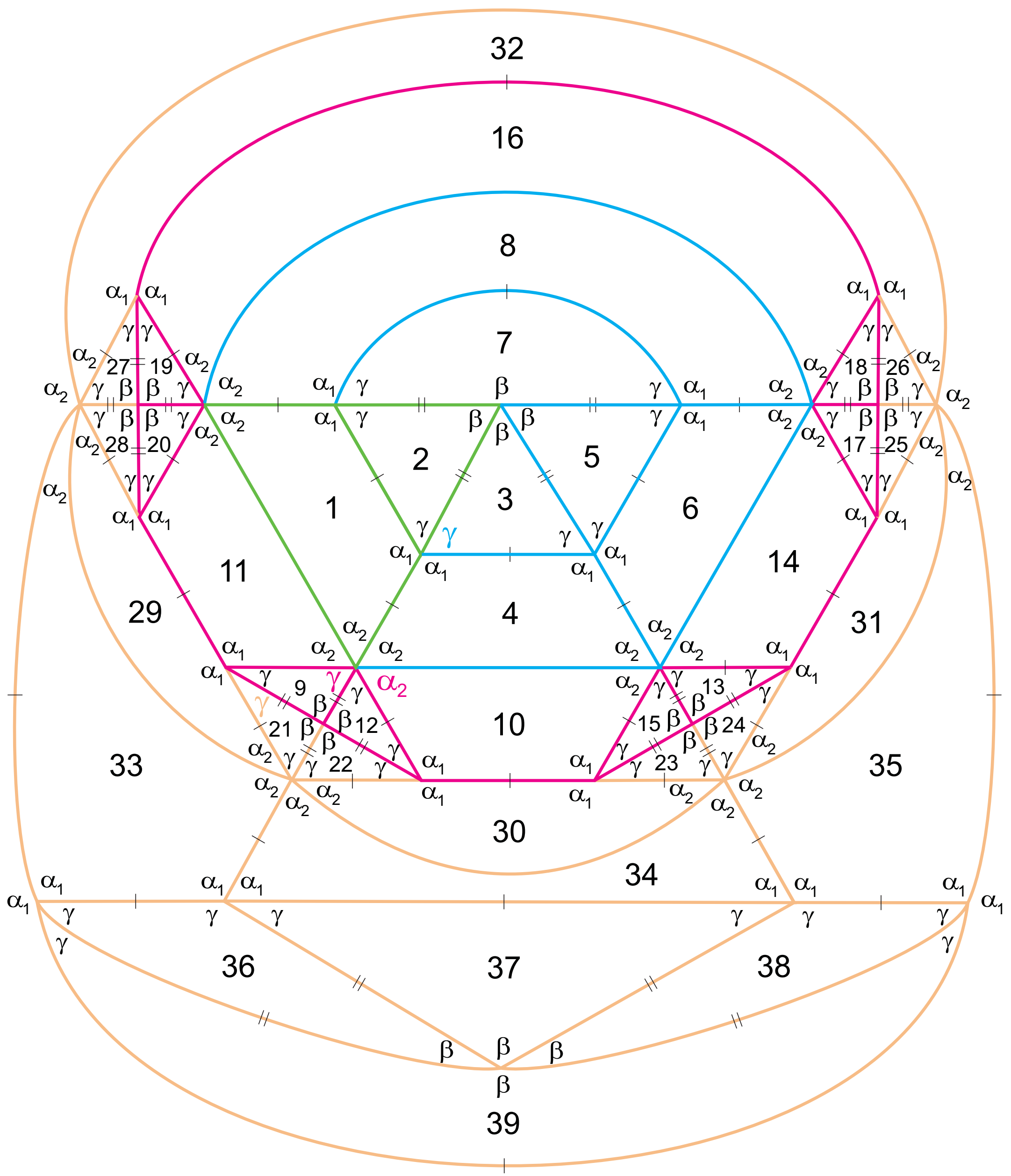

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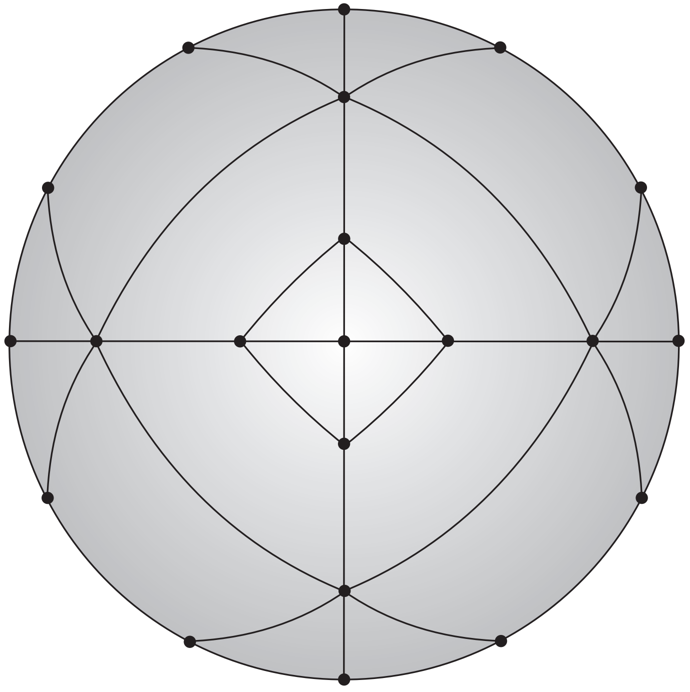

$$M = 24$$

$$N = 16$$

$$|V| = 3$$

$$G(\tau) = C_2 \times D_4$$

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f-Tiling	α_1	α_2	γ	β	$\lvert V\rvert$	M	N	$G(\tau)$
$\mathcal{T}$	$\arccos\frac{1-\sqrt{5}}{2}$	$\frac{\pi}{2}$	$\frac{\pi}{2}$	$\pi-\alpha_1$	3	8	4	C_2
$\mathcal{C}$	$\arccos\frac{-2}{3}$	$\frac{\alpha_1}{2}$	$\pi-\alpha_1$	$\frac{\pi}{2}$	3	24	16	$C_2\times D_4$
$\bar{\mathcal{C}}_\gamma$	$\pi-\gamma$	$\frac{\pi}{3}$	$\left(\frac{\pi}{4},\frac{\pi}{3}\right)$	$\frac{\pi}{2}$	3	24	24	O_h
$\mathcal{R}^k_{\alpha_1}$	$\left(\frac{\pi}{2},\frac{(k+1)\pi}{2k}\right)$	$\frac{\pi}{2}$	$\pi-\alpha_1$	$\frac{\pi}{k}$	3	$4k$	$4k$	$C_2\times D_{2k}$
$\bar{\mathcal{R}}^k_{\alpha_2}$	$\pi-\gamma$	$\left(\alpha_2^k,\frac{2\pi}{k}\right)$	$\frac{\pi}{k}$	$\pi-\alpha_2$	3	$4k$	$2k$	D_{2k}

Table 1: Combinatorial Structure of the Dihedral f-Tilings of S^2 by Isosceles Trapezoids and Isosceles Triangles

- $\alpha_2^k=\pi-\arccos\left(-\cos^2\frac{\pi}{k}\right)$, $k\geq 3$;
- $\lvert V\rvert$ is the number of distinct classes of congruent vertices;
- M and N are, respectively, the number of triangles congruent to T and the number of isosceles trapezoids congruent to Q, used in the dihedral f-tilings;
- $G(\tau)$ is the symmetry group of each tiling $\tau\in\Omega(Q,T)$; by C_n we mean the cyclic group of order n; D_n is the dihedral group of order $2n$; the octahedral group is $O_h\cong C_2\times S_4$ (the symmetry group of the cube).

Thank you!