4th Combinatorics Day - Aveiro, May 14, 2014

In honour of Raul Cordovil

Programme

Room Sousa Pinto
Departamento de Matemática, Universidade de Aveiro

10:30-11:15 Alex Fink (Queen Mary University of London)

Matroids over a ring

Matroids arise in many combinatorial problems of algebraic flavour, which feature lists of vectors over a field. But often one's data are elements in a module over some other ring, and there is more information to be extracted than the field-agnostic linear algebra that the matroid can see. Luca Moci and I have defined the notion of _matroid over a ring_ to expose this extra information. I will discuss two examples of situations where matroids over rings capture extra combinatorics, one related to subtorus arrangements and the other to tropical geometry. I'll also discuss a little bit of their structure theory, and how to generalize the Tutte invariant.

11:15-11:45 Coffee break

11:45-12:15 Inês Soares (INESCC, UC)

Modelling Phylogenetic Trees: a combinatorial approach

A phylogeny is a synopsis of the evolutionary events that derived from a common ancestral genome producing the diversity currently known. During the last decades many approaches have been developed in order to address the problem of phylogeny reconstitution. Nevertheless, the reconstruction of the evolutionary process is entirely dependent on the quality of the sequence comparison step, which remains a great challenge to research community up today due to the complexity of the events that lead to changes on nucleic acid sequences.

Among other methods that we developed, we present here a new algorithm that uses dynamic programming tables and suffix tree structures to automate the extraction of mitochondrial regions and the sequence comparison. We also explain a tentative method based on a construction of Dawson, which depends on the choice of an order of variable positions that should correspond to the order in which the mutations have occurred, to compare and cluster mitochondrial genomes.

This is joint work with António Guedes de Oliveira and António Amorim.

12:15-12:45 Leonor Moreira (CMUP, UP) and **José Luís Fachada (IST)**

"Manual de sobrevivência a um combinatorialista bem conhecido" ou *"Como aturar o único orientador capaz de me aturar"*

12:45-14:30 Lunch

14:30-15:00 Jorge Orestes Cerdeira (CMA, FCT/UNL)

Some applications of combinatorics to ecology

I will describe the use of combinatorial tools to address ecological issues such as characterizing the niche of species and niche overlap.

15:00-15:30 Enide Andrade Martins (CIDMA, UA)

Laplacian and signless Laplacian spectra of graphs having vertex subsets with common neighborhood properties

In this work we present results related with the Laplacian and signless Laplacian spectra of a graph G of order n, with k vertices that share the same neighbors (that is, with k pairwise co-neighbor vertices).

Using a result presented in [2], it is shown that the number of shared neighbors is a Laplacian and a signless Laplacian eigenvalue of G with multiplicity at least $k-1$.

Additionally, considering a connected graph G_k with a vertex set defined by the k pairwise co-neighbor vertices of G, we establish that the Laplacian spectrum of G^k, obtained from G adding the edges of G_k, includes all eigenvalues $l+\beta$ for each nonzero Laplacian eigenvalue β of G_k. It is referred also that the Laplacian spectrum of G overlaps the Laplacian spectrum of G^k in at least $n-k+1$ places. Finally, it is determined the signless Laplacian spectra of G^k (when G_k is regular).

This is joint work with Nair Abreu, D. M. Cardoso, M. Robbiano and B. San Martín.

[1] N.M.M. Abreu, Domingos M. Cardoso, E. A. Martins, M. Robbiano, B. San Martín, On the Laplacian and signless Laplacian spectrum of a graph with k pairwise co-neighbor vertices, Linear Algebra Appl. 437 (2012): 2308-2316.

[2] I. Faria, Permanental roots and the star degree of a graph, Linear Algebra Appl. 64 (1985): 255–265.

15:30-16:00 Domingos Moreira Cardoso (CIDMA, UA)

Star sets, star complements and graphs with convex-qp stability number

Consider a graph G with n vertices and an adjacency eigenvalue λ (simply called an eigenvalue of G). Let P be the matrix of the orthogonal projection of $\mathbb{R}^n$ onto the eigenspace of λ, $\mathcal{E}_G(\lambda)$, with respect to the standard orthonormal basis $\{e_1, \ldots, e_n\}$ of $\mathbb{R}^n$. Then the set of vectors Pe_j $(j = 1, \ldots n)$ spans $\mathcal{E}_G(\lambda)$ and therefore there exists $X \subseteq V(G)$ such that the vectors Pe_j $(j \in X)$ form a basis for $\mathcal{E}_G(\lambda)$. Such a set X is called a *star set* for λ in G or simply a λ-star set of G and $\bar{X} = V(G) \setminus X$ is said a λ-*co-star set* of G, while $G - X = G\left[\bar{X}\right]$ is called a *star complement* for λ in G. If G has m distinct eigenvalues $\mu_1 \geq \cdots \geq \mu_m$, where each eigenvalue μ_i has multiplicity $k_i, i = 1, \ldots, m$ (and then $\sum_{i=1}^{m} k_i = n$), it can be proved that there is a partition $X_1 \cup \cdots \cup X_m$ of $V(G)$ where each part X_i is a μ_i-star set (and then has cardinality k_i). This partition is called a *star partition* of G.

The graphs for which the stability number, that is, the size of a stable set (a set of mutually non-adjacent vertices) of maximum cardinality, can be determined solving a convex quadratic program are called *graphs with convex-qp stability number*, where *qp* means quadratic programming. The graphs with convex-*qp* stability number are called $\mathcal{Q}$-graphs.

In this presentation, several combinatorial properties of $\mathcal{Q}$-graphs, G, are highlighted and a few relations between star complements of the least eigenvalue of G and its maximum stable sets are presented. As a consequence, a simplex-like approach to the recognition of $\mathcal{Q}$-graphs is described.

This is a joint work with Carlos J. Luz.

16:00-16:30 Coffee break

16:30-17:00 Pedro J. Freitas (CELC, UL)

Counting Spectral Radii of Matrices with Positive Entries

We present some results concerning the number of different spectral radii that you can get for matrices with entries in a prescribed set of positive integers. This problem is related to a conjecture by Erdős and Szemerédi about sums and products of elements in a finite set. This is a joint work with José Dias da Silva.

17:00-17:30 António Breda d'Azevedo (CIDMA, UA)

Pseudo-orientability versus orientability

Despite not being a topological property, pseudo-orientability is a combinatorial propriety introduced by Steve Wilson in the eighties that shows interesting resemblances with orientability in respect to maps (cellular embeddings of "multiple" graphs on compact connected surfaces). In this talk we empathize their similarities and differences.

17:30-18:00 Samuel Lopes (CMUP, UP)

Combinatorics gone Weyl

The multiplication and differentiation operators x and d/dx generate the Weyl algebra. More generally, given a nonzero polynomial $h = h(x)$, let y be the operator $h.d/dx$, so that x and y satisfy the commutation relation $[y, x] = h$. The associative and unital algebra generated by x and y is denoted by A_h and it is a subalgebra of the Weyl algebra. For suitable choices of h we obtain the Weyl algebra, the enveloping algebra of the two-dimensional non-abelian Lie algebra, and the Jordan plane.

Some interesting combinatorics emerge from the representation theory of the algebras A_h over fields of arbitrary characteristic. We will discuss these and phrase themin the language of partitions.

This is joint work with Georgia Benkart and Matt Ondrus.

Organizers: Rui Duarte (UA), António Guedes de Oliveira (CMUP, UP) and Olga Azenhas (CMUC, UC)

Sponsors: Departamento de Matemática da Universidade de Aveiro, CIM, CMUP and CIDMA

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