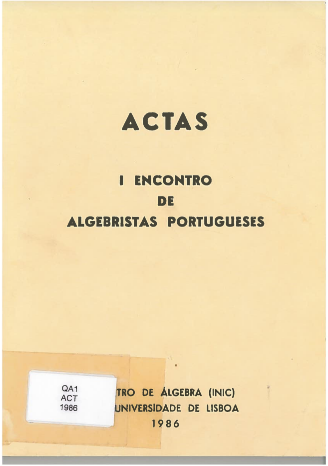
Manual de sobrevivência a um combinatorialista bem conhecido

Maria Leonor Moreira

May 14, 2014

Sobre as Desvantagens de Ser Recta

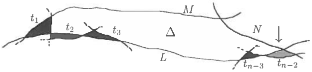


PROGRAMA DO

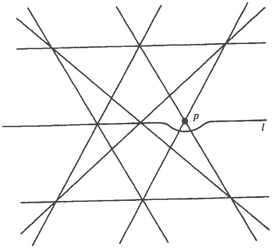
1º ENCONTRO DE ALGEBRISTAS PORTUGUESES

Segunda Feira dia 17 de Fevereiro

9-9.30	Abertura
10-10.45	Margarita Ramalho (Universidade de Lisboa) "Álgebras com reduto na variedade dos Semireticulados ou na dos reticulados distributivos".
11-11.45	Jorge Almeida (Universidade do Minho) "Pseudovariedades de semigrupos".
12-12.45	Gabriela Bordalo (Universidade de Lisboa) "Algumas consequências da teoria da dualidade em reticulados distributivos".
14.30-15.15	Raul Cordovil (Investigador do I.N.I.C.) "Sobre as desvantagens de ser recto".
15.30-16.15	Owen John Brison (Universidade de Lisboa) "Algumas questões em classes de Fitting".
16.30-18.	Mesa Redonda: "Sobre as perspectivas da investigação em Álgebra em Portugal".



Arranjos de Pseudorectas



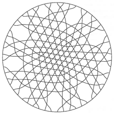
1.

By definition a <u>configuration of pseudolines and points</u> in the Euclidean plane E^2 is a pair $(\mathcal{L}, \mathcal{P})$ constituted by

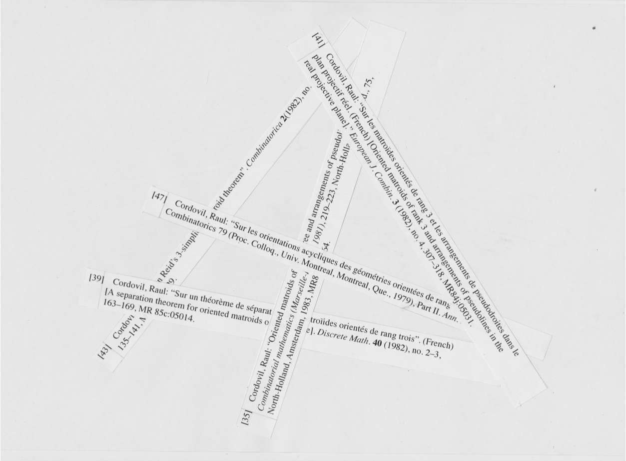
a finite family $\mathcal{L}$ of pseudolines (images of lines by homeomorphism of E^2) and a finite family $\mathcal{P}$ of points such that:

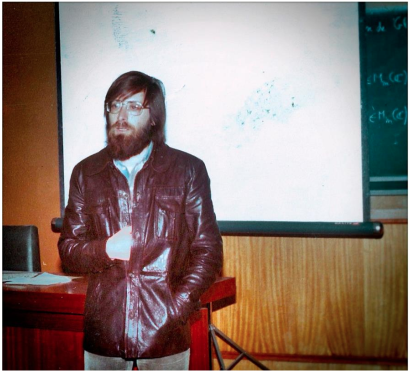
(i) For every pair (p, q) of different points of $\mathcal{P}$ there exists one and only one pseudoline $L \in \mathcal{L}$ containing p and q;

(ii) For every pseudoline $L \in \mathcal{L}$



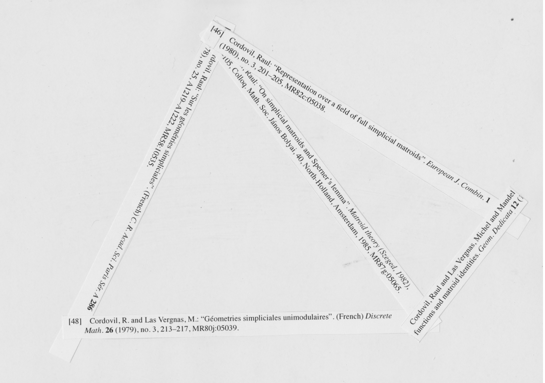
Matroides Orientados de Rank 3:





Nem sempre orientados! . . .

Matroides Simpliciais:



6

Simplicial Matroids

RAUL CORDOVIL AND BERNT LINDSTRÖM

6.1. Introduction

Given a finite set $A=\{a_1,a_2,\ldots,a_n\}$ and an integer k with $0\leqslant k\leqslant n$, let $\binom{A}{k}$ denote the set of all k-element subsets of A. A k-element set will also be called a *k-simplex*, but we must warn the reader that topologists would prefer the name $(k-1)$-simplex since the topological realization has dimension $k-1$. Formal linear combinations of k-simplices in $\binom{A}{k}$ with coefficients from a field F give a vector space $F^{\binom{A}{k}}$ of dimension $\binom{n}{k}$ over F.

For $X\in\binom{A}{k}$ define the *boundary* $\partial X\in F^{\binom{A}{k-1}}$

$$\partial(\varnothing)=0, \tag{6.1}$$

$$\partial(\{a_i\})=0, \tag{6.2}$$

$$\partial(\{a_{i_1},\ldots,a_{i_k}\})=\sum_{j=1}^{k}(-1)^{j-1}\{a_{i_1},\ldots,\hat{a}_{i_j},\ldots,a_{i_k}\},\quad (i_1<\cdots<i_k). \tag{6.3}$$

The roof ˆ over a letter means 'delete it'.

The boundary operation is extended by linearity to all elements of $F^{\binom{A}{k}}$:

$$\partial\left(\sum_{v=1}^{m}c_vX_v\right)=\sum_{v=1}^{m}c_v\partial(X_v),\quad \text{where } c_1,\ldots,c_v\in F. \tag{6.4}$$

The following important property of the boundary operation is left as an easy exercise:

$$\partial^2X=0,\quad X\in F^{\binom{A}{k}},\quad k\geqslant 1 \tag{6.5}$$

6.1.1. Definition. A subset $\{X_1,\ldots,X_m\}\subseteq\binom{A}{k}$ is independent in the *full simplicial matroid* $S_k^A[F]$ if $\partial(X_1),\ldots,\partial(X_m)$ are linearly independent over F. The restriction of $S_k^A[F]$ to a subset $E\subseteq\binom{A}{k}$ is a *k-simplicial geometry (matroid)* if $k\geqslant 2$ (if $k=0$ or 1).

It is easy to prove that the matroids $S_k^A[F]$ and $S_k^B[F]$ are isomorphic when $|A|=|B|=n$. In particular, the linear order of A (used in (6.3)) does not matter. The matroid is therefore also denoted by $S_k^n[F]$, where n is called the *order*.

6.1.2. Example. Consider a finite simple graph with vertex set A and edge set $E\subseteq\binom{A}{2}$. The 2-simplicial geometry on E over a field F is the cycle matroid of the graph (which does not depend on F).

Sometimes it is desirable to order the elements of a simplex in a linear order different from the initial linear order of A. Consider a k-simplex $X=\{a_{i_1},a_{i_2},\ldots,a_{i_k}\}$, where $i_1<i_2<\cdots<i_k$, and assume that $a_{j_1},a_{j_2},\ldots,a_{j_k}$ is a permutation of X. Then we define the *oriented simplex*

$$(a_{j_1},a_{j_2},\ldots,a_{j_k})=\operatorname{sign}\begin{pmatrix}i_1i_2\cdots i_k\\ j_1j_2\cdots j_k\end{pmatrix}\{a_{i_1},\ldots,a_{i_k}\}, \tag{6.6}$$

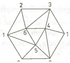
where the sign, $+$ or $-$, depends on the parity of the permutation. One can prove as an exercise

$$\partial(a_{j_1},a_{j_2},\ldots,a_{j_k})=\operatorname{sign}\begin{pmatrix}i_1i_2\cdots i_k\\ j_1j_2\cdots j_k\end{pmatrix}\partial(a_{i_1},\ldots,a_{i_k}). \tag{6.7}$$

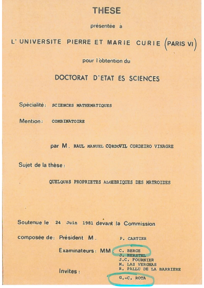
6.1.3. Example. Consider the triangulation of the real projective plane in Figure 6.1. It is easy to verify

$$\begin{aligned}&\partial[(1,2,4)+(1,2,6)+(1,4,3)+(1,5,3)+(2,3,5)+(2,3,6)\\&+(1,6,5)+(2,5,4)+(3,4,6)+(4,5,6)]=2[(1,2)+(2,3)+(3,1)]=0\end{aligned}$$

Figure 6.1. A triangulation of the real projective plane.



Alguns Deuses:



THESE

présentée à

L'UNIVERSITE PIERRE ET MARIE CURIE (PARIS VI)

pour l'obtention du

DOCTORAT D'ETAT ES SCIENCES

Spécialité: SCIENCES MATHEMATIQUES

Mention: COMBINATOIRE

par M. RAUL MANUEL CORDOVIL CORDEIRO VINAGRE

Sujet de la thèse:

QUELQUES PROPRIETES ALGEBRIQUES DES MATROIDES

Soutenue le 24 Juin 1981 devant la Commission

composée de: Président M. P. CARTIER

Examinateurs: MM C. BERGE
J. BERSTEL
J.C. FOURNIER
M. LAS VERGNAS
R. PALLU DE LA BARRIERE

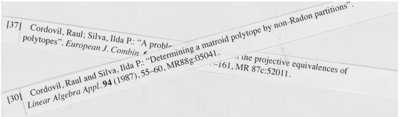
Invites: G.-C. ROTA

Gian Carlo Rota, Branko Grunbaüm, Claude Berge

Família Matemática

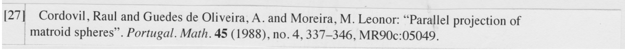
- Michel Las Vergnas
- Folkman , Lawrence, Edmonds, Lindstrom
- Arnaldo Mandel, Komei Fukuda
- P. Duchet, T. Zaslavski, N. White, Y. Hamidoune
- David Forge, Jorge Luís Ramírez Alfonsin, Manoel Lemos

Ilda Perez da Silva

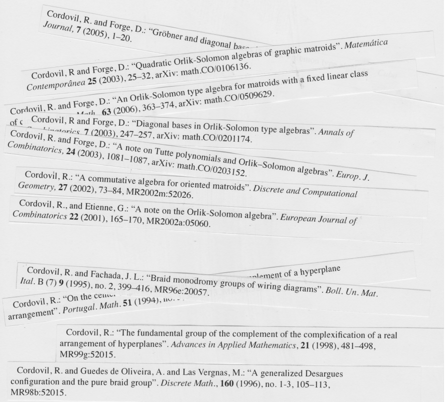

[37] Cordovil, Raul; Silva, Ilda P.: "A proble... the projective equivalences of polytopes". *European J. Combin* ... –161, MR 87c:52011.

[30] Cordovil, Raul and Silva, Ilda P.: "Determining a matroid polytope by non-Radon partitions". *Linear Algebra Appl.* **94** (1987), 55–60, MR88g:05041.

António Guedes de Oliveira


[27] Cordovil, Raul and Guedes de Oliveira, A. and Moreira, M. Leonor: "Parallel projection of matroid spheres". *Portugal. Math.* **45** (1988), no. 4, 337–346, MR90c:05049.

- Protecção
- Exigência



- Sobrevivi . . . . com os não-orientados!


Graph Theory