The 6th Combinatorics Day - Almada, July 14, 2016

Programme

Room Sala de Seminários - Ed. VII
Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa

10:00-11:00 Vincent Pilaud (CNRS & LIX, École Polytechnique):

Permutrees

Permutrees are oriented and labeled trees satisfying certain local conditions around each vertex. They gather under the same roof several combinatorial families, including permutations, binary trees, and binary sequences. The talk will present their combinatorial, geometric and algebraic structure. In particular, we will show:

∗ the permutree lattice, which generalizes the weak order on permutations, the Tamari lattice on binary trees, and the boolean lattice on binary sequences;

∗ the permutreehedron, which generalizes the permutahedron, the associahedron, and the cube;

∗ the permutree Hopf algebra, which generalizes the Malvenuto-Reutenauer Hopf algebra on permutations, the Loday-Ronco Hopf algebra on binary trees, and Solomon's descent Hopf algebra on binary sequences.

This talk is based on a joint work with Viviane Pons (LRI, Université Paris Sud).

11:00-11:30 Coffee break

11:30-12:00 Antonio Macchia (CMUC):

The poset of proper divisibility

We define the order relation given by the proper divisibility of monomials, inspired by the definition of the Buchberger graph of a monomial ideal. From this order relation we obtain a new class of posets. Surprisingly, the order complexes of these posets are homologically non-trivial. We prove that these posets are dual CL-shellable, we completely describe their homology (with integer coefficients) and we compute their Euler characteristic. Moreover this order relation gives the first example of a dual CL-shellable poset that is not CL-shellable. Joint work with Davide Bolognini, Emanuele Ventura and Volkmar Welker.

12:00-12:30 António Leal Duarte (CMUC, UC):

Downer vertices for eigenvalues of trees

12:30-13:00 Manuel Branco (CIMA-UE):

On the enumeration of the set of elementary numerical semigroups with fixed multiplicity, Frobenius number or genus

Let $\mathbb{N}$ denote the set of nonnegative integers. A numerical semigroup is a subset S of $\mathbb{N}$ that is closed under addition, $0 \in S$ and $\mathbb{N}\backslash S$ has finitely many elements. The cardinality of the set $\mathbb{N}\backslash S$ is called the genus of S and it is denoted by $\mathrm{g}(S)$.

For any numerical semigroup S, the smallest positive integer belonging to S (respectively, the greatest does not belong to S) is called the multiplicity (respectively Frobenius number) of S and it is denoted by $\mathrm{m}(S)$ (respectively F) (see [6]). We say that a numerical semigroup S is elementary if $\mathrm{F}(S) < 2\mathrm{m}(S)$.

Given a positive integer g, we denote by $\mathcal{S}(g)$ the set of all numerical semigroups with genus g. The problem of determining the cardinal of $\mathcal{S}(g)$ has been widely treated in the literature (see for example [1], [2], [3], [4], [5] and [7]. Some of these works are motivated by Amorós' conjecture [3] which says the sequence of cardinals of $\mathcal{S}(g)$ for $g = 1, 2, \cdots$ has a Fibonacci behavior. It is still not known in general if for a fixed positive integer g there are more numerical semigroups with genus $g+1$ than numerical semigroups with genus g.

In this talk we give algorithms that allows to compute the set of every elementary numerical semigroups with a given genus g, Frobenius number F and multiplicity m. As a consequence we obtain formulas for the cardinal of these sets. In particular we show that sequence of cardinals of the set of elementary numerical semigroups of genus $g = 0, 1, \ldots$ is a Fibonacci sequence. This is joint work with J.C. Rosales (Universidad de Granada).

[1] V. Blanco, P. A. García-Sánchez and Justo Puerto, Counting numerical semigroups with short generating functions, Int. J. of Algebra and Comput. 21(7), 1217-1235, (2011).

[2] M. Bras-Amorós, Bounds on the number of numerical semigroups of a given genus, J. Pure Appl. Algebra, 213(6), 997-1001 (2008).

[3] M. Bras-Amorós, Fibonacci-like behavior of the number of numerical semigroups of a given genus, Semigroup Forum 76, 379-384 (2008).

[4] S. Elizalde, Improved bounds on the number of numerical semigroups of a given genus, J. Pure Appl. Algebra, 214(10), 1862-1873 (2010).

[5] N. Kaplan, Couting numerical semigroups by genus and some cases a question of Wilf, J. Pure Appl. Algebra, 216(5), 1016-1032 (2012).

[6] J. C. Rosales, P. A. García-Sánchez, "Numerical semigroups", Developments in Mathematics, vol.20, Springer, New York, (2009).

[7] Y. Zhao, Constructing numerical semigroups of a given genus, Semigroup Forum 80(2), 242-254 (2009).

13:00-15:00 Lunch

15:00-15:30 Rosário Fernandes (FCT-UNL):

The degree of a matching

Let $T = (V(T), E(T))$ be a tree. A matching in T is a subset M of $E(T)$ without two edges adjacent in T. Let $G_T = (V(G_T), E(G_T))$ be the matching graph of T where $V(G_T)$ is the set of all matchings in T, and two matchings M and N are adjacent if and only if the union of the sets $(M \setminus N)$ and $(N \setminus M)$ is the nontrivial set of the edges of a path in T. We describe an algorithm for computing the degree of a matching in the matching graph.

15:30-16:00 Carlos Saiago (CMA, FCT-UNL):

Diameter, Seeds and Branch Duplication for Trees

Let T be a tree and $S(T)$ be the set of all real symmetric matrices whose graph is T. In 2002, C.R. Johnson and A. Leal-Duarte have shown that, for each tree T of diameter $d(T)$ (measured as the number of vertices in a longest induced path), any matrix $A \in S(T)$ has, at least, $d(T)$ distinct eigenvalues. We use seeds and branch duplication to show that, when $d(T) < 7$, there is a matrix in $S(T)$ with exactly $d(T)$ distinct eigenvalues. We also discuss the case in which $d(T) = 7$. Joint work with Charles R. Johnson (College of William and Mary, Williamsburg).

16:00-16:30 Coffee break

16:30-17:00 Joel Moreira (Ohio State University):

Monochromatic configurations in finite colorings of $\mathbb{N}$

Is it possible to color the natural numbers with finitely many colors, so that whenever x and y are of the same color, their sum $x + y$ has a different color? A 1916 theorem of I. Schur tells us that the answer is no. In other words, for any finite coloring of $\mathbb{N}$, there exist x and y such that the triple $\{x, y, x + y\}$ is monochromatic (i.e. has all terms have the same color). A similar result holds if one replaces the sum $x + y$ with the product xy, however, it is still unknown whether one can finitely color the natural numbers in a way that no quadruple $\{x, y, x + y, xy\}$ is monochromatic! In this talk I present a recent partial solution to this problem, showing that any finite coloring of the natural numbers yields a monochromatic triple $\{x, x + y, xy\}$.

17:00-17:30 António Malheiro (CMA, FCT-UNL):

Crystallizing the hypoplactic monoid

Crystal graphs, in the sense of Kashiwara, carry a natural monoid structure given by identifying words labelling vertices that appear in the same position of isomorphic components of the crystal. In the particular case of the crystal graph for the q-analogue of the special linear Lie algebra sl_n, this monoid is the celebrated plactic monoid, whose elements can be identified with Young tableaux. The crystal graph and the so-called Kashiwara operators interact beautifully with the combinatorics of Young tableaux and with the Robinson–Schensted correspondence and so provide powerful combinatorial tools to work with them. In a joint work with A. Cain we construct an analogous 'quasi-crystal' structure for the hypoplactic monoid, whose elements can be identified with quasi-ribbon tableaux and whose connection with the theory of quasi-symmetric functions echoes the connection of the plactic monoid with the theory of symmetric functions. This quasi-crystal structure and the associated quasi-Kashiwara operators are shown to interact just as neatly with the combinatorics of quasi-ribbon tableaux and with the hypoplactic version of the Robinson–Schensted correspondence. We studied the interaction of the crystal graph of the plactic monoid and the quasi-crystal graph for the hypoplactic monoid. Finally, the quasi-crystal structure is applied to prove some new results about the hypoplactic monoid. Joint work with Alan Cain (CMA-UNL).

Organizers: Manuel Silva (CMA,UNL), Olga Azenhas (CMUC,UC) and António Guedes de Oliveira (CMUP,FC-UP).

URL: `http://www.mat.uc.pt/~combdays/6thcombday`