The 7th Combinatorics Day - Évora, May 26, 2017

Programme

Room Anfiteatro 1, Colégio Luís António Verney
Universidade de Évora

9:30-10:30 Jorge Ramírez Alfonsín (Université de Montpellier):

Integer points and Ehrhart polynomial of lattice path matroid polytope

The problem of computing the set of integer points inside plane bodies has a long history, and in a more general context can be thought of as a discretization of the usual notion of volume. If P is an integer polytope then the *Ehrhart polynomial* is the number of integer lattice points in the *dilated polytope tP*. This polynomial, in the integer parameter t, encodes the relationship between the volume P and the number of integer points contained in P.

In this talk we focus our attention to *matroid base polytopes*. After giving a gentle introduction on Ehrhart polynomials and matroids, we investigate the number of integer points lying in dilations of *lattice path matroid polytopes*. We first give a combinatorial characterization of such points. We then present a relation with *distributive* and *order* polytopes as well as an explicit formula of the Ehrhart polynomial for some special cases. Finally, a conjecture on the *modularity* of some coefficients (called *h-vector*) will be discussed.

10:30-11:00 Coffee break

11:00-11:30 Rui Duarte (CIDMA, University of Aveiro)

Hyperplane arrangements: between Shi and Ish

We introduce a new family of hyperplane arrangements in dimension $n \geq 3$ that includes both the Shi arrangement and the Ish arrangement. We prove that all the members of this family have the same number of regions which can be bijectively labelled with the Pak-Stanley labelling. In addition, we show that, in the cases of the Shi and the Ish arrangements, the number of labels with *reverse centers* of a given length is equal, and conjecture that the same happens with all of the members of the family. This is joint work with António Guedes de Oliveira.

11:30-12:00 Denise Torrão (University of Évora):

Combinatorial optimization problems

In this talk, we will study the digital semigroups and the bracelet monoids. A digital semigroup D is a subsemigroup of $(\mathbb{N}\backslash\{0\},\cdot)$ such that if $d \in D$ then $\{x \in \mathbb{N}\backslash\{0\} \mid \ell(x) = \ell(d)\} \subseteq D$ with $\ell(n)$ the number of digits of n written in decimal expansion. We compute the smallest digital semigroup containing a set of positive integers. For this, we establish a connection between the digital semigroups and a class of numerical semigroups called LD-semigroups.

Given positive integers $n_1, \ldots, n_p$, we say that a submonoid M of $(\mathbb{N}, +)$ is a $(n_1, \ldots, n_p)$-bracelet if $a+b+\{n_1, \ldots, n_p\} \subseteq M$ for every $a, b \in M \backslash \{0\}$. We explicitly describe the smallest $(n_1, \ldots, n_p)$-bracelet that contains a finite subset X of $\mathbb{N}$. We also present a recursive method that enables us to construct the whole set $\mathcal{B}(n_1, \ldots, n_p) = \{M \mid M$ is a $(n_1, \ldots, n_p) -$ bracelet$\}$. Finally, we study $(n_1, \ldots, n_p)$-bracelets that cannot be expressed as the intersection of $(n_1, \ldots, n_p)$-bracelets properly containing it.

This is joint work with J. C. Rosales (Universidad de Granada - Spain) and M. B. Branco (Universidade de Évora).

J. C. Rosales, M. B. Branco and D. Torrão, *Bracelet monoids and numerical semigroups*, Applicable Algebra in Engeneering, Communication and Computing, (2015), 1-15.

J. C. Rosales, M. B. Branco and D. Torrão, *Sets of integers closed under product and the number of decimal digits*, Journal of Number Theory 147 (2015), 1-13.

12:00-12:30 António Girão (University of Cambridge):

Edge Colourings

Given a graph G with maximum degree Δ we are interested in studying under which conditions is it always possible, to extend any partial proper colouring of the edges of G, to an optimal proper edge colouring of the entire graph. We will talk about some recent results which allow us to do so, and we shall prove that if the distance between any two pre-coloured edges is sufficiently large then such a pre colouring has an extension to a proper edge colouring of G using at most $\Delta + 1$ colours. The talk is based on joint work with Ross Kang.

12:30-15:30 Lunch

15:30-16:30 Martin Rubey (Technische Universität Wien):

On the combinatorics of the symplectic group

The symplectic group $Sp(V)$ on a $2n$-dimensional complex vector space is the group of linear transformations preserving a non-degenerate skew-symmetric bilinear form. The combinatorial study of the representation theory of $Sp(V)$ began in 1937, when Brauer describes the algebra of endomorphisms of a tensor power of V, commuting with the diagonal action of $Sp(V)$, in terms of perfect matchings.

In the 70's and 80's, King, Berele and Sundaram develop a theory of symplectic tableaux. This yields a major result of Sundaram: a combinatorial description of the restriction of a given irreducible representations of the general linear group to the symplectic group.

In the 90's, the theory of crystal graphs is introduced by Kashiwara, which yields insertion algorithms for all classical groups. When specialised to the symplectic group, this theory is incompatible with the one introduced by King, Berele and Sundaram.

In joint work with Westbury, we use Brauer's and Sundaram's results to produce a so called second fundamental theorem of invariant theory: a description in terms of perfect matchings of a basis of the subspace of tensors invariant under the action of $Sp(V)$. The main feature of this basis is that it is invariant under rotation. In particular, this yields a new instance of Reiner, Stanton and White's cyclic sieving phenomenon.

Switching to crystal graphs, in joint work with Pfannerer and Westbury, we show that rotation of the elements in the aforementioned basis coincides with promotion of highest weight words. More generally, we study the action of Henrique's and Kamnitzer's cactus groups on these words, and provide a description in terms of Fomin's growth diagrams.

16:30-17:00 Coffee break

17:00-17:30 Alan J. Cain (CMA, FCT-New University of Lisbon):

Identities in plactic, hypoplactic, sylvester, Baxter, and related monoids

The famous plactic monoid, whose elements can be viewed as Young tableaux and which is connected with many aspects of combinatorics and algebra, is just one of a family of monoids whose elements can be viewed as combinatorial objects. Others monoids include the hypoplactic monoid (the monoid of quasi-ribbon tableaux, connected with the theory of quasi-symmetric functions), the sylvester monoid (binary search trees), and the Baxter monoid (pairs of twin binary search trees, connected with the theory of Baxter permutations).

Jaszuńska & Okniński proved that the Chinese monoid, which is closely related to the plactic monoid, satisfies a non-trivial identity. (An identity is a formal equality $u = v$, where u and v are words over some alphabet of variables, and is non trivial if u and v are not identical as words. It is satisfied by a monoid if every substitution of elements for variables yields equality in the monoid. For example, any commutative monoid satisfies $xy = yx$.) This naturally raises the questions of whether the plactic monoid and the other monoids discussed above satisfy non-trivial identities.

This talk will present new results settling these questions. In particular, the plactic monoid does not satisfy a non-trivial identity, whereas the hypoplactic, sylvester, and Baxter monoids do satisfy non-trivial identities, and it is possible to characterize completely the shortest non-trivial identities they satisfy. The proofs are via the corresponding combinatorial objects. This is joint work with António Malheiro (CMA-UNL).

17:30-18:00 Carlos Florentino (CAMGSD, FC-University of Lisbon)

On the topology of symmetric products

Symmetric products and their topological invariants have been extensively studied in the literature; in particular, MacDonald's formulae for symmetric products of algebraic curves, have been greatly generalized to include many singular cases. Motivated by the case of character varieties of free abelian groups (which turn out to be special cases of symmetric products of diagonalizable groups), we describe a new method of computing the mixed Hodge structures of these and of more general symmetric products, based on the character table of the symmetric group. This is joint work with Jaime Silva.

18:00-18:30 Manuel Silva (CMA, FCT-New University of Lisbon)

Monochromatic paths for the integers

A famous result of van der Waerden states that any finite coloring of the naturals has arbitrarily long monochromatic arithmetic sequences. We will discuss some Ramsey related problems concerning the structure of the sets of differences which can not be avoided. This is a joint work with João Guerreiro and Imre Ruzsa

19:30 Dinner

Organizers: Manuel Branco (CIMA,UE), Olga Azenhas (CMUC,UC) and António Guedes de Oliveira (CMUP,FC-UP).

URL: http://www.mat.uc.pt/∼combdays/7thcombday