

The 8th Combinatorics Day - Porto, June 22, 2018

Programme

Edifício da Matemática, Faculty of Sciences of University of Porto.

9:00-10:00 Drew Armstrong (University of Miami):

Strange Expectations

I will talk about recent work of Marko Thiel and Nathan Williams on “strange expectations” and the “Winnie-the-Pooh” problem, which was motivated by Johnson’s theorem (also known as Armstrong’s conjecture). The original conjecture states that for a, b coprime the average number of boxes in an (a, b) -core partition is $(a+b+1)(a-1)(b-1)/24$. Paul Johnson used Ehrhart theory to show that the average size must be a polynomial of degree 2, from which the result follows by interpolation. Thiel and Williams generalized Johnson’s approach to arbitrary root systems, and explained that the 24 in the denominator comes from the “strange formula” of Freudenthal and de Vries. More recently, Thiel and Williams proved a vastly more general formula for the “average length” of a weight vector in a semisimple Lie algebra representation. I will try to give a birds-eye view of these beautiful results.

10:00-10:30 Coffee break

10:30-11:30 Cédric Lecouvey (Université de Tours):

Combinatorics of generalised exponents

Generalised exponents are important graded multiplicities in representation theory of simple Lie algebras. Notably, they are particular Kazhdan-Lusztig polynomials. In type A, they admit a nice combinatorial description in terms of Lascoux-Schützenberger’s charge statistics on semistandard tableaux. In this talk I will recall their definition and explain how to get similar statistics beyond type A. This will give a combinatorial proof of the positivity of their coefficients but also some other interesting properties. This is a work in collaboration with Cristian Lenart.

11:30-12:00 Jocelyn Lochon (CEAFEL, University of Lisbon):

On the representation theory of the Infinite Unitriangular group

Let $U(q)$ denote the set of infinite nilpotent matrices over the finite field with q elements, called infinite unitriangular group. It is a wild group (not type I), thus the irreducible representations do not admit any reasonable parametrization.

Nevertheless the set of all characters (positive definite, normalized class functions) characterizes up to quasi-equivalence representations of type I and II, therefore, they are our main object of study.

The goal of this talk is to describe a Supercharacter-Theory for $U(q)$. Such description relies on set partitions and some combinatorial relations.

12:00-12:30 Raúl Penaguião (Universität Zürich):

Tree conjecture on graphs and more colourings in polytopes

We will talk about some chromatic invariants in graphs and polytopes, and see what are the consequences of a nice description of the kernel of these chromatic maps in important problems like the tree conjecture. We will see that the most general version of this problem is in the realm of generalised permutahedra, as we will see that graphs and other interesting combinatorial objects can be embedded there.

12:30-15:30 Lunch

15:30-16:30 Pedro Silva (CMUP, University of Porto):

On the Dowling and Rhodes matroids

Dowling and Rhodes defined different lattices on the set of triples (Subset, Partition, Cross Section) over a fixed finite group G . Although the Rhodes lattice is not a geometric lattice, it defines a matroid in the sense of the theory of boolean representable simplicial complexes. This turns out to be the direct sum of a complete matroid with a lift matroid of the complete biased graph over G . As is well known, the Dowling lattice defines the frame matroid over a similar biased graph. This gives a new perspective on both matroids and uncovers connections of matroid theory to the theory of finite semigroups. We also make progress on an important question for these classical matroids: what are the minimal boolean representations and the minimum degree of a boolean matrix representation? This is joint work with Stuart Margolis (Bar Ilan University, Israel) and John Rhodes (University of California at Berkeley, USA).

16:30-17:00 Coffee break

17:00-17:30 Paula Carvalho (CIDMA, University of Aveiro):

Lexicographic polynomials of graphs

For a (simple) graph H and non-negative integers c_0, c_1, \dots, c_d ($c_d \neq 0$), $p(H) = \sum_{k=0}^d c_k \cdot H^k$ is the lexicographic polynomial in H of degree d , where the sum of two graphs is their join and $c_k \cdot H^k$ is the join of c_k copies of H^k . The graph H^k is the k th power of H with respect to the lexicographic product ($H^0 = K_1$). The spectrum (if H is regular) and the Laplacian spectrum (in general case) of $p(H)$ are determined in terms of the spectrum of H and c_k 's. Constructions of infinite families of cospectral or integral graphs are also announced.

N. ABREU, D.M. CARDOSO, P. CARVALHO, C.T.M. VINAGRE: *Spectra and Laplacian spectra of arbitrary powers of lexicographic products of graphs*. Discrete Math., **340** (2017), 3235–3244.

D.M. CARDOSO, P. CARVALHO, P. RAMA, S.K. SIMIĆ, Z. STANIĆ: *Lexicographic polynomials of graphs and their spectra*. Appl. Anal. Discrete Math. **11** (2017), 258–272.

17:30-18:00 Ricardo Mamede (CMUC, University of Coimbra):

A classification of monotone stair ribbons with full Schur support

We consider ribbon shapes whose rows have size at least two, overlapping in at most one column, and with row lengths in monotone order. We give linear inequalities in terms of the partition shape and the overlapping partition, under which such a ribbon Schur function attains all the interval support when expanded in the basis of Schur functions. We then conclude that the strict triangle inequality condition on a connected ribbon, given by M. Gaetz, W. Hardt and S. Sridhar, as a sufficient condition for a ribbon to have full equivalence class implies full Schur support. This is a work in collaboration with O. Azenhas.

Organizers: Olga Azenhas (CMUC, UC), Rui Duarte (CIDMA, UA), António Guedes de Oliveira (CMUP, FC-UP), Samuel Lopes (CMUP, FC-UP).

URL: <http://www.mat.uc.pt/~combdays/8thcombdays>