Commutative Algebras in Fibonacci Categories

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Abstract.

Let $C$ be a braided monoidal category with braiding $c_{X,Y} : X \otimes Y \to Y \otimes X$. An algebra $A$ in $C$ is said to be commutative if $\mu_{c,A,A} = \mu$. If $C$ is also equipped with a natural set of isomorphisms $\theta = \{ \theta_A : A \to A \mid A \in \text{ob}C \}$, subject to suitable axioms, then we call $C$ balanced and the elements of $\theta$ are referred to as (ribbon) twists. A commutative algebra $A$ in a balanced category is then called ribbon if $\theta_A = 1_A$. If $C$ is also rigid then there is a suitable notion of separability for $A$.

Recall that a tortile (rigid and balanced) monoidal category is said to be fusion when it is semi-simple $k$-linear together with a $k$-linear tensor product, finite dimensional hom spaces and a finite number of simple objects (up to isomorphism). We call a fusion category modular if it satisfies a certain non-degeneracy (modularity) condition.

A Fibonacci category is a modular category with the “fusion rule” $X^2 = 1 + X$. By studying Non-negative Integer Matrix (NIM) representations we show that the Fibonacci category and its tensor powers are completely anisotropic; that is, they do not have any non-trivial separable commutative ribbon algebras.

As an application we deduce that a chiral algebra, with the representation category equivalent to a product of Fibonacci categories, is maximal; that is, it is not a proper subalgebra of another chiral algebra.

*Joint work with Alexei Davydov.