

Notions of Möbius inversion

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Abstract.

Every small category \mathbf{A} gives rise to a topological space $|\mathbf{A}|$, called its classifying space or geometric realization. The homotopy type of $|\mathbf{A}|$ depends not only on the underlying graph of \mathbf{A} , but also on its composition. However, the Euler characteristic of $|\mathbf{A}|$ depends only on the underlying graph. This talk can be understood as an exploration of that fact.

The key idea is Möbius inversion. Classical, number-theoretic, Möbius inversion takes place in the poset of positive integers ordered by divisibility. It has been generalized to categories in two different ways. The first, which I will call *fine Möbius inversion*, was introduced independently by Pierre Leroux and John Haigh. It depends on the composition of the category in which it takes place. The second, *coarse Möbius inversion*, was introduced independently by Haigh and the speaker. It does not depend on the composition. I will explain what fine and coarse Möbius inversion are, how they are related to each other, and how they are related to Euler characteristic. The aim is to shed light on the fact that the Euler characteristic of a category is independent of its composition.