On unitary representations of certain quantum groups

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Abstract. *Dagger compact closed categories* have been touted as fundamental structures in the study of quantum computation [1]. *Dagger tortile categories* and *dagger pivotal categories* were introduced in [2]: they differ from dagger compact closed categories only in that the symmetry axiom has been weakened to a lesser or greater extent (respectively).

The unitary representations of a (classical) group g naturally form a dagger compact closed category, $\mathbf{UnRep}(g)$, but linear representations do not: they form a compact closed category, $\mathbf{Rep}(g)$, which lacks an obvious dagger structure. [This is in spite of the fact that, in the case of finite g, there is an equivalence of categories $\mathbf{Rep}(g) \simeq \mathbf{UnRep}(g)$. Because dagger structures are required to be identity-on-objects, they cannot be transported along arbitrary equivalences.]

Quantum groups generalise classical groups in such a way that their categories of "linear representations" form *autonomous categories*: again, these are like compact closed categories, except that they lack symmetry. Therefore one may be led to ask, is it possible to define "unitary representations" of a quantum group g in such a way that they naturally form a dagger pivotal (or, dagger tortile) category?

In full generality, the answer appears to be *no*; however, with a small tweak in the definition of quantum group, the answer becomes *yes*.

References

[1] Samson Abramsky and Bob Coecke. Categorical quantum mechanics. In *Handbook of quantum logic and quantum structures–quantum logic*, pages 261–323. Elsevier/North-Holland, Amsterdam, 2009.

[2] P. Selinger. A survey of graphical languages for monoidal categories. In Bob Coecke, editor, *New Structures for Physics*, volume 813 of *Lecture Notes in Physics*, pages 289–355. Springer Berlin/Heidelberg, 2011.