Categories of quantum theoretic models

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Abstract. There exist two approaches which apply category theory as a principal tool for the foundations of quantum theory: the monoidal dagger-categories approach (developed by Abramsky and Coecke) and the topos-theoretic approach (developed by Isham and Döring). Both approaches have certain virtues: the former is able to characterise the important part of finite-dimensional quantum theory in purely categorical terms, while the latter is able to provide a replacement of boolean spectral valuations with intuitionistic ones, allowing to go beyond the Kochen–Specker no-go theorem for 'realist' interpretations of quantum theory. However, both approaches are currently unable to deal with the infinite-dimensional problems that are generic in quantum field theory and large parts of quantum statistical mechanics, as well as they are unable to deal with the variational optimisation problems that are crucial for quantum estimation and quantum measurement theory. We propose an alternative categorical setting for the foundations of quantum theory that deals with these two problems. On the analytical level, our framework is based on the Falcone–Takesaki non-commutative flow of weights and the Legendre–Fenchel duality in a way which makes it compatible with the convex and algebraic approaches to foundations of quantum theory. On the categorical level, it provides the extension of the categorical framework for statistical models developed by Chentsov, Morse and Sacksteder, McCullagh, and Brøns.