

Categorical Dynamics

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Abstract.

In the (Chicago 1967) setting of a cartesian closed category E of ‘spaces’, with a given pointed ‘infinitesimal’ space T , we call second-order infinitesimal the symmetric square $W = T^2/2!$ whose two axes coalesce to one $\alpha : T \rightarrow W$. As in (Montreal 1997) the cartesian closed category E^α consists of ‘motion laws’ on ‘configuration spaces’ C structured by a prolongation along α , i.e. a section of the restriction $\alpha^* : C^W \rightarrow C^T$. A motion law is geodesic if it commutes with all (point-preserving) endomorphisms λ of T . Newton’s starting point, ‘force laws act on motion laws’, is obtained if we define (Firenze 2011) a force law to be any endomorphism of C^W that fixes α^* . Force laws can be composed, as required in engineering where motion laws must be devised to describe complex situations. In case $C = E^B$, the placements of a body B in an environment E , there are two special kinds of force law (external and internal), the ‘body force’ induced up from a given law on E itself, and the ‘visco-elastic force’ co-induced by a simple constitutive relation on B , namely an endomorphism of $B \times W$ that fixes $B \times T \rightarrow B \times W$. (Thanks to the SDG work of Bell, Bergeron, Bunge, Dubuc, Gago, Heggie, Kock, Lavendhomme, Moerdijk, Reyes, Sawyer, & Wraith, in many E these ‘objective logic’ relations translate, for a dense subcategory of spaces, into classical calculus.)