Butterflies, profunctors and fractions

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Abstract. It is known that monoidal functors between internal groupoids in the category *Grp* of groups constitute the bicategory of fractions of the 2-category *Grpd*(*Grp*) of internal groupoids, internal functors and internal natural transformations in *Grp* with respect to weak equivalences (see [3]). Monoidal functors can be equivalently described by a kind of weak morphisms introduced by B. Noohi in [2] under the name of "butterflies". In order to internalize monoidal functors in a wide context, we introduce the notion of internal butterflies between internal crossed modules in a semi-abelian category $\mathcal{C}$, and we show that they are morphisms of a bicategory $\mathcal{B}(\mathcal{C})$. This bicategory turns out to be equivalent to the bicategory $\mathcal{F}r(\mathcal{C})$ of *fractors* between internal groupoids. We call fractors those internal profunctors characterized by D. Bourn in [1] as the ones whose canonical representation as a span has a fully faithfull, surjective on objects, left leg. For an exact category $\mathcal{C}$, we describe the relationship between $\mathcal{F}r(\mathcal{C})$ and the bicategory of fractions with respect to weak equivalences of the 2-category *Grpd*($\mathcal{C}$) of internal groupoids.

References

[1] D. Bourn, Internal profunctors and commutator theory; applications to extensions, classification and categorical Galois Theory, *Theory and Applications of Categories* **24** (2010) 451–488.

[2] B. Noohi, On weak maps between 2-groups, `arXiv:math/0506313v3` (2005).

[3] E.M. Vitale, Bipullbacks and calculus of fractions, *Cahiers de Topologie et Géométrie Différentielle Catégorique* **51** (2010) 83–113.

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