

On (binary) localic products and localic groups

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Abstract. Despite the discrepancy between topological products and localic products, mimics of entourages (i.e. neighbourhoods of the diagonal) in locales provide a category of uniform locales concretely isomorphic to the usual one defined in terms of covers. By analysing the structure of binary products in the category of locales we obtain the crucial basic facts that produce a transparent construction of the isomorphism. This isomorphism is then applied to the natural uniformities of localic groups ([1], [2]). A surprisingly simple proof of the fact that localic group homomorphisms are uniform is presented, thus providing a natural forgetful functor from the category of localic groups into the category of uniform locales (a fact that mocked the efforts in the language of covers). Even remaking our simple entourage proof to a cover one by translation seems to be rather complex, corroborating the usefulness of the entourage approach.

References

- [1] B. Banaschewski and J. C. C. Vermeulen, *On the completeness of localic groups*, Comment. Math. Univ. Carolinae **40** (1999), 293-307.
- [2] J. R. Isbell, I. Kříž, A. Pultr and J. Rosický, *Remarks on localic groups*, in Categorical Algebra and its Applications (Proc. Int. Conf. Louvain-La-Neuve 1987, ed. by F. Borceux), Lecture Notes in Math. 1348, pp. 154-172.
- [3] J. Picado and A. Pultr, *Entourages, covers and localic groups*, Applied Categorical Structures, to appear (doi: 10.1007/s10485-011-9254-3).

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