

Cellularity, composition, and morphisms of algebraic weak factorization systems

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Abstract. In spaces, some cofibrations are *cellular*—ie, relative cell complexes, formed by repeatedly attaching disks to fill spheres—while others can best be described as retracts of such things. Homotopically all cofibrations are alike, but we have discovered this rather innocuous “cellularity” condition has considerable mileage categorically. In particular, an assignment of cellular structures for the image of the generators of an *algebraic weak factorization system*—ie., a compatible comonad and monad on an arrow category arising from a functorial factorization—uniquely determines an *adjunction*, or more exotically, a *two-variable adjunction of algebraic weak factorization systems*, definitions that are quite rigid. In the proof, the unusual fact that the (co)monad (co)algebras for an algebraic weak factorization system can be composed vertically, forming a double category, plays an essential and somewhat mysterious role. As an application, we can define strict notions of monoidal and enriched algebraic model structures that can nonetheless be shown to exist in examples.