

Topology in Categories of (T, V) -Categories

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Abstract.

It is known that only a small portion of the Joyal-Moerdijk (1994) axioms for open maps suffice to establish a topologically-inspired theory of separation, compactness and perfectness in abstract categories, as given by Penon (1972), Tholen (1999), and Clementino-Giuli-Tholen (2004). Based on recent joint work with Dirk Hofmann (2011), we give a refined approach to this theory that minimizes the conditions on the ambient category but nevertheless yields the desired results when applied to stakeholder categories of topology and analysis. Presenting these in the form $(T, V) - Cat$ (for a *Set*-monad T that has been laxly extended to $V - Rel$, where V is a quantale) we demonstrate in particular the power of a functor $(T, V) - Cat \rightarrow V - Cat$ first introduced by Clementino and Hofmann (2009). For example, proper maps of topological spaces (so that T is the ultrafilter monad, $V = 2$) may be completely characterized in order-theoretic terms, and this simplification process may be exploited more generally.