Generalized Tannakian duality

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Outline

Classical Tannaka duality

Group-like objects

Categories equipped with suitable structures

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Reconstruction problem

Can a group-like object be reconstructed from its category of representations?

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Recognition problem

Which categories are equivalent to categories of representations for some group-like object?

Tannaka duality for Hopf algebras over fields

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then there exists a Hopf algebra H such that $\mathscr{A} \simeq \mathrm{Rep}(H)$.

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There is a biadjunction between k-linear categories over Vect_k and coalgebras.

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A profunctor (also known as distributor or module) $\mathscr{A} \nrightarrow \mathscr{B}$ is a cocontinuous functor $[\mathscr{A}^{\mathrm{op}}, \mathscr{V}] \to [\mathscr{B}^{\mathrm{op}}, \mathscr{V}]$. The category of profunctors is denoted by $\mathbf{Prof}(\mathscr{V})$.

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Observation

Coalgebras are precisely comonads $\mathscr{I} \nrightarrow \mathscr{I}$ in $\mathbf{Prof}(\mathscr{V})$.

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Proof.

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Observation

The Cauchy completion of $\mathscr{I}$ is the full subcategory of dualizable objects in $\mathscr{V}$.

Finding the right environment

Question

Can we characterize $\mathrm{Comod}(C)$ in terms of a 2-categorical universal property in $\mathbf{Prof}(\mathscr{V})$?

Tannaka-Krein objects

Definition

- A coaction of a comonad $c\colon B \to B$ is a morphism $v\colon A \to B$, together with a 2-cell $\rho\colon v \Rightarrow c.v$, compatible with the comonad structure.

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- Every map coaction is isomorphic to $v.f$ for some map f.
- For all maps f and *all 1-cells* g, whiskering with v induces a bijection between 2-cells $g \Rightarrow f$ and morphisms of coactions $v.g \to v.f$.

Tannaka-Krein objects in $\mathbf{Prof}(\mathscr{V})$

Definition

Let C be a cocontinuous comonad on $[\mathscr{B}^{\mathrm{op}}, \mathscr{V}]$. A Cauchy comodule of C is a comodule whose underlying object lies in $\overline{\mathscr{B}}$.

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Theorem (S.)

The forgetful functor $\mathrm{Rep}(C) \to \overline{\mathscr{B}}$ is a Tannaka-Krein object in $\mathbf{Prof}(\mathscr{V})$

Tannakian biadjunction

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Theorem (S.)

If $\mathscr{M}$ is a 2-category with Tannaka-Krein objects, then the functor

$$L\colon \operatorname{Map}(\mathscr{M})/B \to \mathbf{Comon}(B)$$

given by $w \mapsto w.\overline{w}$ has a right biadjoint $\operatorname{Rep}(-)$ (which sends a comonad c to the Tannaka-Krein object of c).

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- This theorem does not require the full strength of the definition of Tannaka-Krein objects.

Monoidal structure on the slice category

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Proposition

The above assignment endows $\mathrm{Map}(\mathscr{M})/B$ with the structure of a monoidal 2-category.

Convolution monoidal structure

Let $\mathscr{M}$ be a monoidal 2-category, let (A, d, e) be a pseudocomonoid in $\mathscr{M}$, and let (B, m, u) be pseudomonoid in $\mathscr{M}$.

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The convolution product $f \star g$ of two 1-cells $f, g \in \mathscr{M}(A, B)$ is given by

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Proposition

Let (B, m, u) be a map pseudomonoid in $\mathscr{M}$. Then $(B, \overline{m}, \overline{u})$ is a pseudocomonoid, and the convolution product on $\mathscr{M}(B, B)$ lifts to the category $\mathbf{Comon}(B)$ of comonads on B.

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A monoid in $\mathbf{Comon}(B)$ is precisely a monoidal comonad.

The Tannakian biadjunction is monoidal

Theorem (S.)

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Thus $L(w) \star L(w') \cong L(w \bullet w')$.

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Corollary

If $\mathscr{M}$ is (braided, sylleptic) monoidal, then the Tannakian biadjunction lifts to (braided, symmetric) pseudomonoids.

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If A and B are autonomous map pseudomonoids, and $w\colon A \to B$ is a strong monoidal map, then $L(w) = w.\overline{w}$ is a Hopf monoidal comonad.

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3. whenever the cokernel of $w(f)$ is finitely generated projective, then the cokernel of f exists and is preserved by w.

Then there exists a Hopf algebroid (H, B) and an equivalence $\mathscr{A} \simeq \mathrm{Rep}(H, B)$.

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Thanks!