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Properties of $\Sigma^{\Sigma^{(-)}}$-algebras in Equ

The category Equ of equilogical spaces, introduced in [2], provides a useful locally cartesian closed extension of the category Top_0 of T_0-spaces and continuous maps; the embedding of T_0-spaces is full and preserves all the existing locally cartesian closed structure ([5, 6]). The Sierpinski space Σ, consisting of two elements, one open and one closed, is the open-subset classifier, *i.e.* given a T_0-space S, for every T_0-space X, a continuous map $f\colon X \to \Sigma^S$ determines precisely an open subset of $X \times S$; nevertheless, Σ^S is an equilogical space which need not be a topological space. In other words, Equ allows one to work with T_0-spaces as if they *were* a cartesian closed category.

The monad of the double power of Σ was considered in different settings in many papers, see for example [3, 4]. This led us to analyze the self-adjoint functor $\Sigma^{(-)}\colon \mathrm{Equ} \to \mathrm{Equ}^{op}$ and the monad of the double power of Σ on the category of equilogical spaces. Interestingly, in [1], this double power monad on Equ gives an intrinsic description of the soberification of a T_0-space.

In this talk we investigate the category of the algebras for the double power monad of Σ on Equ, pointing out a connection with the category of frames and frame homomorphisms; in particular, we recall how the structure of $\Sigma^{\Sigma^{(-)}}$-algebra on an equilogical space gives rise to a frame on the set of its global sections. We then focus on some particular subcategories of Equ: the category of continuous lattices, the category of algebraic lattices and Top_0 itself, restricting the double power monad to each of them and analyzing the algebras in each case. Finally, we introduce a full subcategory REqu of Equ, involving algebraic lattices and equivalence relations on them, and use an algebraic approch to determine the $\Sigma^{\Sigma^{(-)}}$-algebras in REqu and their relationship with spatial frames.

References:

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*Joint work with Giuseppe Rosolini.