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Coherently closed tangent categories
and the link between SDG and the differential λ-calculus

Type theories for smooth maps have been independently studied by two schools of thought with different motivations. The first is synthetic differential geometry (SDG) [1, 2, 3]. Here, one uses the type theory of a topos to reason about microlinear spaces. The motivation is the development of a rigorous foundation for synthetic arguments used in differential geometry. The second is the differential λ-calculus, an explicit type theory for reasoning in smooth models of linear logic (Köthe sequence spaces, convenient vector spaces) [4, 5, 6, 7]. The motivation is to provide a syntax for resource sensitive proofs/computations [8].

The type theories are linked in a simple manner: categorical models of either are always tangent categories [9, 10]. Surprisingly, they are more intimately related as well. This talk will develop a direct relationship between Euclidean vector bundles in SDG, and the differential λ-calculus.

More generally, we will show that the differential bundles over a fixed base B (the analog of vector bundles in a tangent category) of any *coherent, locally cartesian closed tangent category* are a model of the differential λ-calculus. Thus, in SDG, the local reasoning in the category of vector bundles over B is captured by the differential λ-calculus. Having an explicit logic for vector bundles makes lifting certain parts of classical differential geometry, for example, Lagrangian systems and symplectic geometry, more direct.

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