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Hopf formulae for Tor

A Hopf formula expresses a homology object in terms of a projective presentation, its kernel and certain (generalised) commutators. The first such formula, for second group homology, was given by Hopf in 1942. Over the last 13 years or so, Everaert, Gran, Van der Linden and others have developed Hopf formulae in more general categorical contexts. One of these general contexts is that of a semi-abelian category with a Birkhoff subcategory where the reflector factors through a protoadditive functor. In that generality, some elements of the Hopf formula are necessarily very abstract. With Tim Van der Linden and Guram Donadze, I am studying the special situation of subvarieties of categories of R-modules. It can be seen using properties of algebraic theories that every such subvariety is again a category of modules. Here we can find explicit and easy formulations of the generalised commutators. Since the reflector in this situation turns out to be tensoring, the resulting homology functors are Tor functors. Through these fairly simple formulations we obtain new ways of calculating, for example, homology of Lie algebras, and Hochschild homology of an associative unital algebra. More generally, our results apply to any abelian Birkhoff subcategory of any semi-abelian variety, using a factorisation through the abelian core.

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