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An embedding theorem for regular Mal'tsev categories

Barr's embedding theorem for regular categories [1] provides, for each small regular category $\mathcal{C}$, a full and faithful embedding $\mathcal{C} \hookrightarrow \mathsf{Set}^{\mathcal{P}}$ preserving finite limits and regular epimorphisms into a presheaf category. In the abelian context, Lubkin's embedding theorem [6] states that any small abelian category $\mathcal{A}$ admits a faithful conservative exact embedding $\mathcal{A} \hookrightarrow \mathsf{Ab}$ into the category of abelian groups. The aim of this talk is to present similar embedding theorems in the non-abelian context, and in particular for regular Mal'tsev categories.

A regular category is a Mal'tsev category when the composition of equivalence relations on each object is commutative, or equivalently, when each reflexive relation is an equivalence relation [2, 3]. I shall show a construction of a particular regular Mal'tsev locally finitely presentable category $\mathcal{M}$ in terms of (partial) operations and identities. This category can be thought of as a 'representing Mal'tsev category' in the sense that the following embedding theorem holds [4]: each small regular Mal'tsev category $\mathcal{C}$ admits a faithful conservative embedding $\mathcal{C} \hookrightarrow \mathcal{M}^n$ which preserves finite limits and regular epimorphisms. Here, n is the (cardinal) number of subobjects of the terminal object 1 of $\mathcal{C}$. This embedding theorem allows one to prove results about finite limits and regular epimorphisms for regular Mal'tsev categories using elements and operations in an (essentially) algebraic way.

Similar embedding theorems also hold for regular unital, regular strongly unital, regular subtractive and n-permutable categories [5].

References:

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*Joint work with Zurab Janelidze.