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Integration in tangent categories

Since the turn of the 21^{st} century, the theory of differential categories has lead to significant progress in the abstract understanding of differentiation in a variety of settings. In particular, tangent categories [3, 1], which come equipped with a tangent functor, provide an axiomatic setting for differential geometry, while cartesian differential categories [4], which come equipped with a differential combinator, axiomatizes the directional derivative. Recently there has been effort put into studying the axiomatization of integration and antiderivatives in the various differential category settings [5]. In this talk we will introduce the notion of integration in a tangent category, which involves integrating linear bundle morphisms between differential bundles [2]. We will also discuss integration for cartesian differential categories and show the relation with tangent category integration. With this, we will be able to formalize a number of properties of integration, such as Fubini's theorem, the Fundamental Theorems of Calculus, integration of forms, and Stoke's theorem for tangent categories.

References:

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