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*Duality theorems for essential inclusions of Grothendieck toposes*

An inclusion of toposes is said to be essential if the inverse image functor has an extra left adjoint. In their paper of 1989 entitled ‘On the Complete Lattice of Essential Localizations’ [1], Kelly and Lawvere gave a characterisation of essential inclusions of Grothendieck toposes, and also established a duality between essential inclusions of presheaf toposes and idempotent ideals on the respective base category. In the talk we will see extensions of both of these results, which appear in the speaker’s Ph.D. thesis [2].

We shall analyse the cases where the extra left adjoint of an essential inclusion has specific exactness properties, such as preserving finite limits or preserving finite products, and exhibit the corresponding characterisations. We shall give a final answer to the question posed in the 100th PSSL in Cambridge regarding the stability of essential inclusions under pullback, and explain how it relates to their stability under the inclusion-surjection factorisation.

We shall also generalise the aforementioned duality result of Kelly and Lawvere from presheaf toposes to general Grothendieck toposes, and show how when applied to the special case of localic toposes one can find another proof for the result of Johnstone and Moerdijk [3] which characterises local geometric morphisms between toposes over a topological space.

REFERENCES:

- [1] G. Kelly and F. Lawvere, On the complete lattice of essential localizations, *Bull. Soc. Math. Belg.* vol. Ser. A, XLI(2) (1989) 289–319.
- [2] G.F. Lima, Cartesian and finite-product-preserving essential inclusions of Grothendieck toposes, *Ph.D. thesis – University of Cambridge* (2016).
- [3] P. Johnstone and I. Moerdijk, Local maps of toposes, *Proc. London Math. Soc.* vol. 3, no. 58 (1989) 281–305.