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Algebraic duality and the abstract functional analysis of distribution monads

Given a commutative ring S in a suitable category $\mathscr{V}$, the familiar process of dualization of S-modules leads to a form of abstract functional analysis, in terms of which certain measure and distribution monads can be studied [5, 6]. Generalizing from S-modules to $\mathscr{T}$-algebras for a suitable $\mathscr{V}$-enriched algebraic theory $\mathscr{T}$ on a system of arities $\mathscr{J}$ [4], we arrive at the notions of *functional-analytic context* and *functional distribution monad* [1], which capture several kinds of measures, probability measures, distributions, and filters, as well as certain hyperspaces of closed subsets.

In this talk, we study a notion of dualization with respect to a given object S of an arbitrary $\mathscr{J}$-algebraic $\mathscr{V}$-category $\mathscr{A}$, leading to a general study of dualities between algebraic categories. Building on an insight of Freyd, we show that every dual adjunction $\Delta \dashv \nabla : \mathscr{B}^{op} \to \mathscr{A}$ between $\mathscr{J}$-algebraic $\mathscr{V}$-categories is given by dualizing with respect to a *bifold algebra* S , i.e. an object of $\mathscr{V}$ equipped with a pair of commuting algebra structures for specified $\mathscr{J}$-theories $\mathscr{T}$ and $\mathscr{S}$. Calling such adjunctions *$\mathscr{J}$-algebraic dualities*, we characterize those whose inducing bifold algebra S exhibits $\mathscr{T}$ and $\mathscr{S}$ as *commutants* of each other [2, 3], leading to the notion of *stable* $\mathscr{J}$-algebraic duality. This yields an equivalent formulation of functional-analytic contexts as certain stable $\mathscr{J}$-algebraic dualities. We discuss several examples of $\mathscr{J}$-algebraic dualities, functional-analytic contexts, and functional distribution monads.

References:

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