The canonical intensive quality of a pre-cohesive topos

In the context of Lawvere’s Axiomatic Cohesion [1], an essential and local geometric morphism \( p : \mathcal{E} \rightarrow \mathcal{S} \) between toposes is **cohesive** if

i) \( p_! : \mathcal{E} \rightarrow \mathcal{S} \) preserves finite products.

ii) (“Continuity”) for every \( E \in \mathcal{E} \) and \( S \in \mathcal{S} \) the induced morphism \( p_!(E^{(p^*S)}) \rightarrow (pE)^S \) is an isomorphism.

iii) (“Nullstellensatz”) the canonical map \( \theta : p_* \rightarrow p! \) is epi.

Without the continuity condition ii), we refer to \( p : \mathcal{E} \rightarrow \mathcal{S} \) as **pre-cohesive** [3]. For any pre-cohesive \( p : \mathcal{E} \rightarrow \mathcal{S} \), [1] constructs the associated canonical intensive quality as the full subcategory \( \mathcal{L} \) of \( \mathcal{E} \) of those objects \( X \) for which \( \theta_X : p_*X \rightarrow p!X \) is an isomorphism. We call \( \mathcal{L} \) the Leibniz category associated to \( p \).

In this talk we will review some of the basic properties of the category \( \mathcal{L} \), we will give elementary constructions of the left and right adjoints of the inclusion functor \( \mathcal{L} \rightarrow \mathcal{E} \), and we will determine sufficient conditions for a pieces preserving geometric morphism [2] \( g : \mathcal{F} \rightarrow \mathcal{E} \) between two pre-cohesive toposes over \( \mathcal{S} \) to restrict to a geometric morphism between the corresponding Leibniz categories.

Furthermore, we will produce a subcanonical site for the Leibniz category determined by the cohesive site over sets of piecewise linear functions constructed in [4].

References:


*Joint work with Matías Menni.*