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Topological groupoids and exponentiability

We consider exponentiable objects and morphisms in the 2-category $\mathrm{Gpd}(\mathcal{C})$ of internal groupoids in a category $\mathcal{C}$ with finite coproducts when $\mathcal{C}$ is: (1) finitely complete, (2) cartesian closed, and (3) locally cartesian closed. The examples of interest include (1) topological spaces, (2) compactly generated spaces, and (3) sets, respectively. It is well known that if $\mathcal{C}$ is the category of sets or any topos, then $G \to B$ is exponentiable in $\mathrm{Gpd}(\mathcal{C})/B$ if and only it is a fibration. We will see that the sufficiency of this condition extends to the case when $\mathcal{C}$ is merely finitely complete if each $G_i \to B_i$ is exponentiable in $\mathcal{C}$, where the G_i and B_i are the objects of objects, objects of morphisms, and objects of composable pairs, for $i = 0, 1, 2$, respectively. When $\mathcal{C}$ is the category of compactly generated spaces, this includes the case where each B_i is weakly Hausdorff.

We will also consider pseudo-exponentiable morphisms in the pseudo-slice categories $\mathrm{Gpd}(\mathcal{C})//B$. Since the latter is the Kleisli category of a monad T on the strict slice over B, we can apply a general theorem from [1] which states that if TY is exponentiable in a 2-category $\mathcal{K}$, then Y is pseudo-exponentiable in the Kleisi category $\mathcal{K}_T$. Consequently, we will see that $\mathrm{Gpd}(\mathcal{C})//B$ is pseudo-cartesian closed, when $\mathcal{C}$ is the category of compactly generated spaces and each B_i is weakly Hausdorff, and $\mathrm{Gpd}(\mathcal{C})$ is locally pseudo-cartesian closed when $\mathcal{C}$ is the category of sets or any locally cartesian closed category.

References:

[1] S.B. Niefield, Exponentiability in homotopy slices of Top and pseudo-slices of Cat, *Theory and Applications of Categories* 19 (2007), 4–18.

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