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The orbifold construction for join restriction categories

It is natural to describe a geometric object by an atlas in a variety of contexts. We are most familiar with manifolds; other examples include orbifolds [2] and foliations. A drawback of this approach is the difficulty of defining maps between the objects: for manifolds, for instance, one needs to first find a suitable refinement of the atlas on the domain object and then follow this by a map of atlases. A similar approach can be used to define maps between orbifolds, but here it is further complicated by the difficulty in determining whether two such maps are the same or not, since unlike the case for manifolds, two distinct maps between orbifolds may induce the same map on the underlying spaces.

Grandis [1] introduced an elegant way around the need for refinements for manifolds. His idea was to view a manifold as a type of diagram of charts with partial maps between them, indexed by a chaotic category (with precisely one arrow between any two objects). Maps between such diagrams are then given by a matrix of partial maps satisfying certain properties. This makes the category of manifolds easier to work with, and also allows us to define manifold objects for any join restriction category.

We generalize this construction to a more orbifold-like context where one needs a more complicated indexing category, with parallel arrows and non-identity automorphisms. We introduce an orbifold construction for join restriction categories. We define orbifolds using inverse categories as indexing categories, and then defining orbifold objects as linking functors from our index category into a given join restriction category B. Maps between these orbifolds will be (isomorphism classes of) a particular type of modules over the base category B. With this approach, we can define a category of orbifold objects for B which is again a join restriction category. We will show that this construction defines a monad on the category of join restriction categories, and discuss how our construction relates to the standard orbifolds.

References:

[1] M. Grandis, Cohesive categories and manifolds, *Annali di Matematica Pura ed Applicata* 157 (1990) 199–244.

[2] I. Satake, On a generalisation of the notion of manifold, *Proc. Nat. Sci. U.S.A* 42 (1956) 359–363.

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