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A general limit lifting theorem for 2-dimensional monad theory

An important question for monad theory is the possibility of lifting limits along the forgetful functor of the category of algebras. The article I will present [1] deals with this subject within the theory of 2-categories. For strict morphisms of algebras, it is well known [2] that all limits lift. However, as it is usually the case, the relevant notions for its applications are the weaker pseudo and lax notions, and for these notions it is no longer the case that all limits lift. There are in the literature significant lifting results [3], [4] for the 2-categories of pseudo and lax morphisms of algebras.

Let T be a 2-monad on a 2-category $\mathcal{K}$, and let Ω be a family of 2-cells of $\mathcal{K}$. We consider in [1] the notion of a lax morphism such that its structural 2-cell is in $\mathcal{K}$. There are three *distinguished* families of 2-cells which can be considered in any 2-category, and by doing so we recover the notions of lax, pseudo and strict T-algebra morphisms. We also consider a notion of weak limit which is a *weighted* version of Gray's cartesian quasi-limits [5], and define what it means for such a limit to be compatible with another family of 2-cells. These concepts allow to state and prove a limit lifting theorem which unifies and generalizes the results of [2], [3], [4] above.

Another result of [1], which simplifies the proof of our theorem by allowing to consider only conical limits, is the following: every (weighted) weak limit can be expressed as a conical weak limit, with respect to the same family of 2-cells. By considering the three distinguished families of 2-cells as above, our result yields previously known weighted-as-conical results for lax limits, σ-limits [6] and Street's result [7], expressing any strict limit as a cartesian quasi-limit.

References:

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