

J. Adámek *

Technical University of Braunschweig

Free Completions of Categories

Every category K is known to have a free completion PK under colimits. A number of properties of K transfer to PK (e.g., completeness or cartesian closedness). For complete categories K with biproducts we prove that PK is cartesian closed. We further prove that PK always a pretopos, but seldom a topos: assuming the existence of copowers, it is only a topos for essentially small categories. We also study the question whether PK is (co)wellpowered. The answer is affirmative for “set-like” categories. But for a number of categories K the answer turns out to be negative.

A closely related topic is the free completion ΣK of K under coproducts. We prove that it is cartesian closed iff the category K is cartesian multi-closed, i.e. for every object A the endofunctor of multiplication by A is a left multi-adjoint. From this we conclude that ΣK is a topos iff K has a subobject classifier and is cartesian multi-closed.

*Joint work with J. Rosický.