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*Day convolution, stable  $\infty$ -operads and Goodwillie calculus*

For a pointed compactly-generated  $\infty$ -category  $C$ , we construct a stable non-unital  $\infty$ -operad  $I_C^\otimes$  that represents the Goodwillie derivatives of the identity functor on  $C$ . The  $\infty$ -operad  $I_C^\otimes$  is constructed from the Day convolution of two symmetric monoidal structures: the pointwise smash product of spectrum-valued functors on  $C$ ; and the smash product of spectra. There is a relative version of the construction of  $I_C^\otimes$  that assigns to any functor  $F : C \rightarrow D$  a bimodule over the  $\infty$ -operads  $I_C^\otimes$  and  $I_D^\otimes$  that consists of the derivatives of  $F$ .

Our main examples are the  $\infty$ -category of based spaces, in which case  $I_C^\otimes$  recovers a spectral version of the Lie operad, and the  $\infty$ -category of spectrum-valued algebras over a stable non-unital  $\infty$ -operad  $O^\otimes$ , in which case we can identify the  $\infty$ -operad  $I_C^\otimes$  with a completion of  $O^\otimes$  itself.

This work is the foundation for a future project to classify Goodwillie towers of functors between  $\infty$ -categories using pro-operads and their bimodules.