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*Hypergraph categories as cospan algebras*

Hypergraph categories are symmetric monoidal categories in which every object is equipped with a special commutative Frobenius monoid, in a way coherent with the monoidal product. Morphisms in a hypergraph category can hence be represented by string diagrams in which strings can branch and split: diagrams that are reminiscent of electrical circuits. As such they provide a framework for formalising the syntax and semantics of circuit-type diagrammatic languages.

This structure has been independently rediscovered many times, first by Carboni and Walters under the name well-supported compact closed category, and in contexts as diverse as concurrency theory, databases, signal flow graphs and linear algebra, graph rewriting, and circuit theory. Nonetheless, despite its utility, the definition has the perhaps intriguing property that hypergraph structure does not transfer along equivalence of categories.

In this talk I will motivate the definition by providing an alternative conceptualisation of a hypergraph category as a lax symmetric monoidal functor

$$F\colon (\mathrm{Cospan}(\mathrm{FinSet}_\Lambda), +) \to (\mathrm{Set}, \times),$$

where $\Lambda$ is some fixed set, and $\mathrm{FinSet}_\Lambda$ is the category where objects are finite sets $X$ equipped with a function $X \to \Lambda$. We call such functors *cospan algebras*. In particular, the category of cospan algebras is equivalent to the category of strict hypergraph categories.

References:

[1] A. Carboni, Matrices, relations, and group representations, *Journal of Algebra*, 136.2 (1991).

[2] B. Fong, The algebra of open and interconnected systems, DPhil thesis, University of Oxford (2016).

[3] B. Fong and D. I. Spivak, Constructing hypergraph categories, *in preparation.*

[*]Joint work with David Spivak.