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Magnitude

Magnitude is a numerical invariant of enriched categories. It unifies many invariants of size from across mathematics, including cardinality, volume, dimension and Euler characteristic. The study of magnitude has spread out in unexpected directions:

- theorems on the geometric content of magnitude have called upon some very sophisticated analysis (Barceló, Carbery, Gimperlein, Goffeng, Meckes);
- magnitude was the springboard for a newly rigorous and systematic theory of diversity, particularly applicable in biological settings (Cobbold, Meckes);
- magnitude has now been categorified to a theory of magnitude homology; thus, magnitude homology is to magnitude as topological homology is to Euler characteristic (Hepworth, Shulman, Willerton).

As well as giving an overview of all this, I will take some time to discuss the role of category theory in the development and which parts (to bend a phrase of Lawvere) come from 'taking enriched categories seriously'.

References:

[1] Juan Antonio Barceló and Anthony Carbery, On the magnitudes of compact sets in Euclidean spaces, *American Journal of Mathematics* 140 (2018), 449–494.

[2] Heiko Gimperlein and Magnus Goffeng, On the magnitude function of domains in Euclidean space, arXiv:1706.06839 (2017).

[3] Richard Hepworth and Simon Willerton, Categorifying the magnitude of a graph, *Homology, Homotopy and Applications* 19 (2017), 31–60.

[4] Tom Leinster, The magnitude of metric spaces, *Documenta Mathematica* 18 (2013), 857–905.

[5] Tom Leinster and Christina Cobbold, Measuring diversity: the importance of species similarity, *Ecology* 93 (2012), 477–489.

[6] Tom Leinster and Mark Meckes, Maximizing diversity in biology and beyond, *Entropy* 18 (2016), no. 88.

[7] Tom Leinster and Mark Meckes, The magnitude of a metric space: from category theory to geometric measure theory, in Nicola Gigli (ed.), *Measure Theory in Non-Smooth Spaces*, De Gruyter Open (2017), 156–193.

[8] Tom Leinster and Michael Shulman, Magnitude homology of enriched categories and metric spaces, arXiv:1711.00802 (2017).