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Zero, and some other ‘infinitesimal’ levels of a cohesive topos

Let $p : \mathcal{E} \rightarrow \mathcal{S}$ be a pre-cohesive geometric morphism (in the sense of [2]). I will show that, under a mild hypothesis, if \mathcal{S} is Boolean then the ‘points’ functor $p_* : \mathcal{E} \rightarrow \mathcal{S}$ is a Unity and Identity for the subcategories $\text{Dec}(\mathcal{E}) \rightarrow \mathcal{E}$ and $\mathcal{E}_{\neg\neg} \rightarrow \mathcal{E}$, of decidable objects and of $\neg\neg$ -sheaves, making them Adjointly Opposite.

If time permits I will also discuss ‘infinitesimal’ dense levels $l : \mathcal{L} \rightarrow \mathcal{E}$; i.e. those such that the composite $\mathcal{L} \rightarrow \mathcal{E} \rightarrow \mathcal{S}$ is a quality type. I will outline some recent work in collaboration with F. Marmolejo which relates these levels and the concept of *Birkhoff object* introduced in [1]. In particular, I will show that, in certain concrete cases, the largest ‘infinitesimal’ level of \mathcal{E} exists and the separated objects with respect to it coincide with Birkhoff objects. These examples also allow us to further illustrate how the definition of Birkhoff object relates with Birkhoff’s Theorem.

REFERENCES:

- [1] Lawvere, F. W., Birkhoff’s Theorem from a geometric perspective: A simple example, *Categ. Gen. Algebr. Struct. Appl.* 4 (2016) 1–8.
- [2] Lawvere, F. W. and Menni, M., Internal Choice holds in the discrete part of any cohesive topos satisfying stable connected codiscreteness, *TAC* 30 (2015) 909–932.